Optimal Power Allocation Between Training and Data for MIMO Two-Way Relay Channels

Xiaofeng Li, Cihan Tepedelenlioğlu, *Member, IEEE*, and Habib Senol, *Member, IEEE*

***Abstract*—Power allocation between training and data in MIMO two-way relay systems is proposed, which takes into consideration both the symmetric and asymmetric cases of the two sources. For the former, we present a closed form for the optimal ratio of data energy to total energy, which is suitable for the single antenna case as well, and can be simplified when the number of antennas is large. We also show that the achievable rate is a monotonically increasing function of the data time. Concerning the asymmetric case, we prove that the difference of the two SNRs is either a concave or convex function of the energy ratio, depending on the imbalance between the two sources. Using this, the minimum SNR between the two sources is maximized.**

***Index Terms*—Power allocation, training and data, asymmetric case, two-way relay channel, mimo.**

I. Introduction

Two-way relay (TWR) systems can significantly extend coverage and increase throughput by reducing the needed time slots for one round of information exchange between two source nodes. To improve the energy efficiency, some works [1]–[3] investigate the power allocation between the three nodes, assuming the sources have perfect channel state information (CSI). References [4]–[6] consider power allocation in the presence of channel estimation error. References [4] and [5] only focus on power allocation between nodes rather than between training and data. The latter is also considered in [6], with no closed form expressions for the optimal training power.

In this work, we study the power allocation between training and data for the MIMO TWR scenario. First we consider the symmetric case, in which the same number of antennas and same transmit power are assumed. To the best of our knowledge, we are the first to derive a closed form expression of the optimal ratio of data energy to training energy, denoted as β, to maximize the achievable rate through maximizing the signal-to-noise ratio (SNR) of the data phase in TWR. Unlike the point-to-point case which leads to a quadratic equation, the optimal β is found by solving a fourth order equation in TWR. Note that the closed form expression also applies for the single antenna case. When the number of antennas at the sources grows larger, the fourth order equation can be reduced to a quadratic equation. Data time and power have an impact on the achievable rate. We show that the achievable rate is a monotonically increasing function of the data time.

The results above can be extended to the asymmetric case as well. To this end we define a power allocation parameter θ, which represents the imbalance between the two source links. In [4] and [6], θ is optimized as $\theta = 0.5$, which corresponds to the symmetric case. However, the premise that the two links are identical may not always hold, due to geographic reasons and power limitations. Our strategy is that for a given θ, we maximize the minimum of the two SNRs at the sources with respect to β, by showing that the difference of the two SNRs is either a concave or convex function of $\beta \in (0, 1)$, depending on θ.

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II. System Model

We consider a half-duplex TWR system with two source nodes $\mathbb{S}_1$ and $\mathbb{S}_2$ and one relay node $\mathbb{R}$, which uses Amplify-and-Forward (AF). We adopt the two time slot protocol, where in the first time slot, $\mathbb{S}_1$ and $\mathbb{S}_2$ transmit data to $\mathbb{R}$ simultaneously, while in the second time slot, $\mathbb{R}$ amplifies and broadcasts its received data to both the source nodes. The number of antennas at $\mathbb{S}_1$, $\mathbb{S}_2$ and $\mathbb{R}$ are M_1, M_2 and N respectively. The channels are assumed to be quasi-static flat fading. The channels from $\mathbb{S}_1$ to $\mathbb{R}$ and from $\mathbb{S}_2$ to $\mathbb{R}$ are $\boldsymbol{H}_1$ and $\boldsymbol{H}_2$, respectively. We also assume channel reciprocity holds, i.e., the channels from $\mathbb{R}$ to $\mathbb{S}_1$ and $\mathbb{R}$ to $\mathbb{S}_2$ are $\boldsymbol{H}_1^T$ and $\boldsymbol{H}_2^T$ respectively. Both $\boldsymbol{H}_1$ and $\boldsymbol{H}_2$ have zero-mean unit-variance independent complex-Gaussian entries. The training scheme is composed of the following two phases and each of the phases has two equal length time slots.

In the training phase, both sources transmit training symbols to the relay over T_τ symbol intervals at the first time slot. The relay scales the superimposed signal by an $N \times N$ diagonal matrix $\boldsymbol{A} = \alpha \boldsymbol{I}$ and then broadcasts the superimposed training signal at the second time slot. The received training signal at $\mathbb{S}_1$ and the power constraints for the training symbols are

$$\begin{aligned} \boldsymbol{Y}_{1\tau} &= \alpha_\tau \sqrt{\frac{\rho_{1\tau}}{M_1}} \boldsymbol{S}_{1\tau} \boldsymbol{P} + \alpha_\tau \sqrt{\frac{\rho_{2\tau}}{M_2}} \boldsymbol{S}_{2\tau} \boldsymbol{Q} \\ &\quad + \alpha_\tau \boldsymbol{Z}_{\mathrm{R}\tau} \boldsymbol{H}_1^T + \boldsymbol{Z}_{1\tau} \\ \operatorname{tr}\left(\boldsymbol{S}_{1\tau} \boldsymbol{S}_{1\tau}^H\right) &= M_1 T_\tau, \ \operatorname{tr}\left(\boldsymbol{S}_{2\tau} \boldsymbol{S}_{2\tau}^H\right) = M_2 T_\tau, \end{aligned} \tag{1}$$

where $\boldsymbol{S}_{1\tau}$ and $\boldsymbol{S}_{2\tau}$ are matrices of training symbols sent by $\mathbb{S}_1$ and $\mathbb{S}_2$ respectively, $\rho_{i\tau}$ is the transmit power of source node i during the training phase and the entries of the noise matrices $\boldsymbol{Z}_{\mathrm{R}\tau}$ and $\boldsymbol{Z}_{1\tau}$ are independent, additive, white, and Gaussian (AWGN) with zero mean and unit variance. We define the matrices to be estimated as $\boldsymbol{P} := \boldsymbol{H}_1 \boldsymbol{H}_1^T$ and $\boldsymbol{Q} := \boldsymbol{H}_2 \boldsymbol{H}_1^T$ for $\mathbb{S}_1$. Note that for TWR channel estimation the composite channels $(\boldsymbol{P}, \boldsymbol{Q})$ are estimated; while $(\boldsymbol{H}_1, \boldsymbol{H}_2) \to (\boldsymbol{P}, \boldsymbol{Q})$ is a lossy transformation, $(\boldsymbol{P}, \boldsymbol{Q})$ is sufficient for detection of $\mathbb{S}_1$'s data. The scale factor α_τ is chosen as

$$\alpha_\tau = \sqrt{\frac{\rho_{\mathrm{R}}}{(\rho_{1\tau} + \rho_{2\tau} + 1)N}}, \tag{2}$$

and satisfies the power constraint ρ_{R} at the relay.

In the data phase, the length of the time slots is defined as T_d. The transmission is the same as the training phase. The received data signal at $\mathbb{S}_1$ and power constraints are

$$\begin{aligned}\boldsymbol{Y}_{1d} = \alpha_d\sqrt{\frac{\rho_{1d}}{M_1}}\boldsymbol{S}_{1d}\boldsymbol{P} + \alpha_d\sqrt{\frac{\rho_{2d}}{M_2}}\boldsymbol{S}_{2d}\boldsymbol{Q} \\ + \alpha_d\boldsymbol{Z}_{\mathrm{R}d}\boldsymbol{H}_1^T + \boldsymbol{Z}_{1d},\end{aligned} \tag{3}$$

$$\mathrm{E}\left[\mathrm{tr}\left(\boldsymbol{S}_{1d}\boldsymbol{S}_{1d}^H\right)\right] = M_1T_d,\ \mathrm{E}\left[\mathrm{tr}\left(\boldsymbol{S}_{2d}\boldsymbol{S}_{2d}^H\right)\right] = M_2T_d,$$

where $\boldsymbol{S}_{1d}$ and $\boldsymbol{S}_{2d}$ are matrices of data symbols, ρ_{id} is the transmit power during the data phase of source node i, $\boldsymbol{Z}_{\mathrm{R}d}$ and $\boldsymbol{Z}_{id}$ are similarly defined as the noise matrices in the training phase, α_d is the power scaling factor at $\mathbb{R}$, and $T_\tau + T_d = T$.

Let $(\hat{\boldsymbol{P}}, \hat{\boldsymbol{Q}})$ be the estimate of $(\boldsymbol{P}, \boldsymbol{Q})$ and $\tilde{\boldsymbol{P}}$ and $\tilde{\boldsymbol{Q}}$ are the residual error of $\boldsymbol{P}$ and $\boldsymbol{Q}$ respectively, where $\boldsymbol{P} = \hat{\boldsymbol{P}} + \tilde{\boldsymbol{P}}$, $\boldsymbol{Q} = \hat{\boldsymbol{Q}} + \tilde{\boldsymbol{Q}}$. The MMSE estimators are $\hat{\boldsymbol{P}} = \boldsymbol{U}\boldsymbol{Y}_{1\tau}$ and $\hat{\boldsymbol{Q}} = \boldsymbol{V}\boldsymbol{Y}_{1\tau}$. Matrices $\boldsymbol{V}$ and $\boldsymbol{U}$ represent the linear transformation of the received signal to estimate $\boldsymbol{Q}$ and $\boldsymbol{P}$, and are given by

$$\begin{aligned}\boldsymbol{U} = \frac{1}{\alpha_\tau}\sqrt{\frac{M_1}{\rho_{1\tau}}}\boldsymbol{S}_{1\tau}^H\Bigg(\frac{\left(\alpha_\tau^2N+1\right)M_1^2}{\alpha_\tau^2(M_1+1)N\rho_{1\tau}}I_{T_\tau} \\ + \boldsymbol{S}_{1\tau}\boldsymbol{S}_{1\tau}^H + \frac{M_1^2\rho_{2\tau}}{(M_1+1)M_2\rho_{1\tau}}\boldsymbol{S}_{2\tau}\boldsymbol{S}_{2\tau}^H\Bigg)^{-1},\end{aligned} \tag{4}$$

and $\boldsymbol{V}$ is similarly obtained as $\boldsymbol{U}$.

According to [7] and [8], the MSE can be further reduced by carefully choosing the training matrices such that

$$\boldsymbol{S}_{1\tau}^H\boldsymbol{S}_{1\tau} = T_\tau\boldsymbol{I}_{M_1},\ \boldsymbol{S}_{2\tau}^H\boldsymbol{S}_{2\tau} = T_\tau\boldsymbol{I}_{M_2},\ \boldsymbol{S}_{1\tau}^H\boldsymbol{S}_{2\tau} = \boldsymbol{0}. \tag{5}$$

Thus, the traces of error covariance matrices are minimized, and the training structures are optimal for both nodes, due to the channel symmetry of the two source nodes.

III. Power Allocation Between Training and Data

A. Symmetric Case

We now discuss how much power and time should be devoted to the training phase to maximize the achievable rate of the data phase. At the foremost we optimize the power allocation for any pair of T_τ and T_d. Then we discuss the influence of T_d on the achievable rate. Based upon the aforementioned optimal structures of the two training sequences (5), we have

$$\mathrm{E}\left[\|\tilde{\boldsymbol{P}}\|_F^2\right] = \mathrm{E}\left[\mathrm{tr}(\tilde{\boldsymbol{P}}\tilde{\boldsymbol{P}}^H)\right] = \frac{d_3M_1N(M_1+1)}{d_1T_\tau(M_1+1)/M_1+d_3}, \tag{6}$$

where $d_1 = \alpha_\tau^2\rho_{1\tau}N/M_1$, $d_2 = \alpha_\tau^2\rho_{2\tau}N/M_2$ and $d_3 = \alpha_\tau^2N + 1$. The case of $\mathrm{E}[\|\tilde{\boldsymbol{Q}}\|_F^2]$ is similar, and is given by

$$\mathrm{E}\left[\|\tilde{\boldsymbol{Q}}\|_F^2\right] = \frac{d_3NM_1M_2}{d_2T_\tau + d_3}. \tag{7}$$

Using the orthogonal principle, $\mathrm{E}[\|\hat{\boldsymbol{Q}}\|_F^2]$ can be obtained as

$$\mathrm{E}\left[\|\hat{\boldsymbol{Q}}\|_F^2\right] = \frac{d_2NM_1M_2T_\tau}{d_2T_\tau + d_3}. \tag{8}$$

Then the achievable rate of $\mathbb{S}_1$ can be expressed as [9]

$$R_1 = \mathrm{E}\left[\frac{T_d}{T}\log\left(\det\left(\boldsymbol{I} + \bar{\gamma}_1\frac{\bar{\boldsymbol{Q}}^H\bar{\boldsymbol{Q}}}{M_2}\right)\right)\right], \tag{9}$$

where $\bar{\gamma}_1$ is the effective average SNR shown by (10) at the bottom of the page, $\bar{\boldsymbol{Q}} = \frac{1}{\sigma_{\hat{Q}}}\hat{\boldsymbol{Q}}$ is the normalized channel and $\sigma_{\hat{Q}}^2 = \mathrm{E}\left[\|\hat{\boldsymbol{Q}}\|_F^2\right]/M_2$.

Recall that we have the following relation between power and time:

$$\rho T = \rho_\tau T_\tau + \rho_d T_d, \tag{11}$$

where ρT is the given total energy. Define β as the ratio of data energy to the total energy, so that

$$\rho_\tau T_\tau = (1-\beta)\rho T,\ \ \rho_d T_d = \beta\rho T. \tag{12}$$

When the two sources have different powers, we define θ as the ratio of the power of $\mathbb{S}_2$ to the total power. Thus,

$$\begin{aligned}\rho_{2d} = \theta\rho_d,\ \ \rho_{1d} = (1-\theta)\rho_d, \\ \rho_{2\tau} = \theta\rho_\tau,\ \ \rho_{1\tau} = (1-\theta)\rho_\tau.\end{aligned} \tag{13}$$

We first consider the symmetric case, where $\theta = 0.5$, $M_1 = M_2 = M$, $\rho_{1\tau} = \rho_{2\tau} = \frac{1}{2}\rho_\tau$ and $\rho_{1d} = \rho_{2d} = \frac{1}{2}\rho_d$.

Proposition 1: For any fixed pair of T_τ and T_d, the optimal β that maximizes R_1 is given by the solution to (14)

$$\begin{aligned}a_1a_2\beta^4 + 2a_1b_2\beta^3 + (3a_1c_2 + b_1b_2 - c_1a_2)\beta^2 \\ + 2b_1c_2\beta + c_1c_2 = 0.\end{aligned} \tag{14}$$

Proof: Since the power ρ_τ and ρ_d can only affect R_1 through the effective SNR $\bar{\gamma}_1$, maximizing R_1 is equivalent to maximizing $\bar{\gamma}_1$. Plugging (12) into (10), it becomes:

$$\bar{\gamma}_1 = \frac{a_1\beta^3 + b_1\beta^2 + c_1\beta}{a_2\beta^2 + b_2\beta + c_2}, \tag{15}$$

where

$$a_1 = \alpha_\tau^4\alpha_d^2N^3\rho^3T^3(M+1)$$

$$b_1 = -2a_1 - 2\alpha_\tau^2\alpha_d^2N^2M^2\rho^2T^2\left(\alpha_\tau^2N+1\right)$$

$$c_1 = a_1 + 2\alpha_\tau^2\alpha_d^2N^2M^2\rho^2T^2\left(\alpha_\tau^2N+1\right)$$

$$a_2 = 2N^2(M+1)T^2\alpha_\tau^2\rho^2\left(NT_d\alpha_d^2\left(1+\frac{1}{N\alpha_d^2}\right) - 2M\left(\alpha_\tau^2N+1\right)\right)$$

$$\begin{aligned}b_2 = 4NT\rho\left(M\alpha_d^2\left(\alpha_\tau^2N+1\right) - NT_d\alpha_\tau^2\alpha_d^2\left(1+\frac{1}{N\alpha_d^2}\right)\right) \\ \times\left(NT(M+1)\alpha_\tau^2\rho + M(2M+1)\left(\alpha_\tau^2N+1\right)\right)\end{aligned}$$

$$\begin{aligned}c_2 = 2NT_d\alpha_d^2\left(1+\frac{1}{N\alpha_d^2}\right)\left(2M^2\left(\alpha_\tau^2N+1\right) + N(M+1)T\alpha_\tau^2\rho\right) \\ \left(2M\left(\alpha_\tau^2N+1\right) + NT\alpha_\tau^2\rho\right).\end{aligned}$$

The optimal β that maximizes $\bar{\gamma}_1$ can be found by $\frac{\partial\bar{\gamma}_1}{\partial\beta} = 0$, which yields (14).

Equation (14) shows that the exact solution of the optimal β^* can be obtained by solving the fourth order equation, analytically or numerically. One can check these roots and choose the one in (0,1) which yields the highest $\bar{\gamma}_1$. Though equations (10)

$$\bar{\gamma}_1 = \frac{\rho_{2d}\mathrm{E}\left[\|\hat{\boldsymbol{Q}}\|_F^2\right]}{\rho_{2d}\mathrm{E}\left[\|\tilde{\boldsymbol{Q}}\|_F^2\right] + \rho_{1d}\frac{M_2}{M_1}\mathrm{E}\left[\|\tilde{\boldsymbol{P}}\|_F^2\right] + M_1M_2\left(N + 1/\alpha_d^2\right)} = \frac{\rho_{2d}d_2T_\tau}{\rho_{2d}d_3 + \rho_{1d}d_3\frac{(M_1+1)(d_2T_\tau+d_3)}{(M_1+1)d_1T_\tau+M_1d_3} + \left(1+\frac{1}{\alpha_d^2N}\right)(d_2T_\tau+d_3)} \tag{10}$$

and (14) are derived for node $\mathbb{S}_1$, β^* is optimal for both sources due to the symmetry of the two sources.

If we consider the case $M \gg 1$, the second term in the denominator of (10) becomes $\rho_{1d}d_3$. Thus, (15) can be simplified as

$$\begin{aligned}\bar{\gamma}_1 &= \frac{\rho_d d_2 T_\tau}{2\rho_d d_3 + 2\left(1 + 1/\left(\alpha_d^2 N\right)\right)(d_2 T_\tau + d_3)} \\ &= \left(\frac{\rho T N \alpha_\tau^2}{4 T_d M}\right) \frac{-\beta^2 + \beta}{a_3\beta + b_3}, \end{aligned} \tag{16}$$

where

$$\begin{aligned} a_3 &= \frac{\alpha_\tau^2 N + 1}{T_d} - \frac{1}{2M}\left(\alpha_\tau^2 N + \frac{\alpha_\tau^2}{\alpha_d^2}\right) = \frac{a_4}{T_d} - b_4, \\ b_3 &= \left(1 + \frac{1}{\alpha_d^2 N}\right)\left(\frac{\alpha_\tau^2 N + 1}{\rho T} + \frac{\alpha_\tau^2 N}{2M}\right). \end{aligned} \tag{17}$$

Taking $\partial\bar{\gamma}_1/\partial\beta = 0$ again, we arrive at the quadratic equation

$$a_3\beta^2 + 2b_3\beta - b_3 = 0. \tag{18}$$

If $a_3 = 0$, we have $\frac{\alpha_\tau^2 N+1}{T_d} = \frac{1}{2M}\left(\alpha_\tau^2 N + \frac{\alpha_\tau^2}{\alpha_d^2}\right)$, the optimal power allocation ratio is $\beta^* = \frac{1}{2}$. In such a case,the total energy is distributed equally between training and data.

If $a_3 \neq 0$, β^* is a root of (18), and the closed form expression is given as

$$\beta^* = \frac{-b_3 + \sqrt{b_3^2 + a_3 b_3}}{a_3}. \tag{19}$$

It can be verified using (17) that (19) is between 0 and 1. Thus we have the expression of β that maximizes $\bar{\gamma}_1$ for all the cases.

For simplicity, we further consider the high SNR case where we have $\rho_\tau, \rho_d \gg 1$ and $\rho_{1\tau} = \rho_{2\tau} = \rho_{\mathrm{R}} = 0.5\rho_\tau$. Then (2) becomes $\alpha_\tau^2 \approx \frac{0.5\rho_\tau}{(0.5\rho_\tau + 0.5\rho_\tau)N} = \frac{1}{2N}$. Similarly we have $\alpha_d^2 \approx \frac{1}{2N}$. Equation (19) can be simplified to

$$\beta^* = \frac{-\left(\frac{3}{\rho T} + \frac{1}{2M}\right) + \sqrt{\left(\frac{3}{\rho T} + \frac{1}{T_d}\right)\left(\frac{3}{\rho T} + \frac{1}{2M}\right)}}{\frac{1}{T_d} - \frac{1}{2M}}. \tag{20}$$

For $a_3 \neq 0$, ($T_d \neq 2M$), as shown by (20), β^* will decrease when M grows larger, with fixed T_d. This indicates that with increased number of antennas at the sources, more energy should be allocated into the training phase.

Equation (19) can be simplified when $M_1 = M_2 = N = 1$, which is the single antenna case in TWR. When $\theta = 0.5$ (symmetric powers for both sources), our results of the optimal β^* coincide with the numerical results provided by [6], which considered the case of $M = N = 1$.

Given the optimal β which is a function of T_d, we now discuss how to choose T_τ and T_d.

Proposition 2: Given the optimal β, R_1 is a monotonically increasing function of T_d. The maximum value of T_d is $T - 2M$.

Proof: Let λ be an arbitrary non-zero eigenvalue of $\frac{\bar{Q}^H\bar{Q}}{M_2}$ ($\lambda > 0$), from (9) we have

$$R_1 \geq \frac{M_2}{T}\mathrm{E}\left[T_d \log(1 + \lambda\bar{\gamma}_1)\right] \tag{21}$$

Taking the derivative of (21) with respect to T_d yields

$$\frac{\partial R_1}{\partial T_d} \geq \frac{M_2}{T}\mathrm{E}\left[\log(1 + \lambda\bar{\gamma}_1) + \frac{T_d}{1 + \bar{\gamma}_1}\frac{\partial\bar{\gamma}_1}{\partial T_d}\right]. \tag{22}$$

We discuss the case of $a_3 < 0$. The other cases have similar arguments and the same results [9]. First we rewrite $\bar{\gamma}_1$ by plugging β^* in (16) as follows:

$$\bar{\gamma}_1 = \frac{\rho T N \alpha_\tau^2}{4M}\frac{1}{b_4 T_d - a_4}\left(\sqrt{\eta} - \sqrt{\eta - 1}\right)^2, \tag{23}$$

where $\eta = -b_3/a_3$ and T_d is involved in η. After some manipulation, we have

$$\frac{\partial\bar{\gamma}_1}{\partial T_d} = \frac{\bar{\gamma}_1 b_4}{b_4 T_d - a_4}\left(\frac{a_4\sqrt{\eta}}{b_4 T_d\sqrt{\eta - 1}} - 1\right) \tag{24}$$

Substituting (24) into (22), then

$$\begin{aligned}\frac{\partial R_1}{\partial T_d} &\geq \frac{M_2}{T}\mathrm{E}\Bigg[\log(1 + \lambda\bar{\gamma}_1) \\ &\qquad - \frac{\lambda\bar{\gamma}_1}{1+\lambda\bar{\gamma}_1}\frac{b_4 T_d}{b_4 T_d - a_4}\left(1 - \frac{a_4\sqrt{\eta}}{b_4 T_d\sqrt{\eta-1}}\right)\Bigg] \quad (25)\\ &\geq \frac{M_2}{T}\mathrm{E}\left[\log(1 + \lambda\bar{\gamma}_1) - \frac{\lambda\bar{\gamma}_1}{1 + \lambda\bar{\gamma}_1}\right], \quad (26)\end{aligned}$$

where

$$0 < \frac{b_4 T_d}{b_4 T_d - a_4}\left(1 - \frac{a_4\sqrt{\eta}}{b_4 T_d\sqrt{\eta - 1}}\right) < 1 \tag{27}$$

The first inequality in (27) can be shown by substituting all the coefficients into the middle term of (27). To prove the second inequality, one can upper bound the middle term of (27) by replacing $\sqrt{\eta - 1}$ with $\sqrt{\eta}$.

Using the inequality $\log(1 + x) - \frac{x}{1+x} \geq 0$ for all $x \geq 0$, on (26) we have $\partial R_1/\partial T_d \geq 0$ and R_1 is a monotonically increasing function of T_d. Thus to maximize R_1, T_d should be chosen as its maximum value. Note that to obtain meaningful estimates of the channels, $T_\tau \geq 2M$ is required in the TWR system to ensure as many measurements as unknowns. Therefore, the choice of $T_\tau = 2M$ and $T_d = T - 2M$ maximizes R_1. This concludes the proof.

B. Asymmetric Case

For the asymmetric case, as the sources have different powers, the formulas for the effective SNRs at the two sources are different. In this case, it cannot be guaranteed that the optimal β for one source is still optimal for the other, and there is a trade-off between the two sources. Without loss of generality, we maximize the smaller one of the two average SNRs. We still use $T_d = T - 2M$ here. The effective average SNRs for $\mathbb{S}_1$ and $\mathbb{S}_2$ are defined as $\bar{\gamma}_1$ and $\bar{\gamma}_2$ respectively, and are given by

$$\bar{\gamma}_1 = \left(\frac{\rho T N \alpha_\tau^2}{T_d M_2}\right)\frac{\theta^2(-\beta^2 + \beta)}{a_{31}\beta + b_{31}}, \tag{28}$$

$$\bar{\gamma}_2 = \left(\frac{\rho T N \alpha_\tau^2}{T_d M_1}\right)\frac{(1-\theta)^2(-\beta^2 + \beta)}{a_{32}\beta + b_{32}}, \tag{29}$$

where

$$\begin{aligned} a_{31} &= \frac{\alpha_\tau^2 N + 1}{T_d} - \frac{\theta}{M_2}\left(\alpha_\tau^2 N + \frac{\alpha_\tau^2}{\alpha_d^2}\right), \\ b_{31} &= \left(1 + \frac{1}{\alpha_d^2 N}\right)\left(\frac{\alpha_\tau^2 N + 1}{\rho T} + \frac{\theta}{M_2}\alpha_\tau^2 N\right), \\ a_{32} &= \frac{\alpha_\tau^2 N + 1}{T_d} - \frac{1-\theta}{M_1}\left(\alpha_\tau^2 N + \frac{\alpha_\tau^2}{\alpha_d^2}\right), \\ b_{32} &= \left(1 + \frac{1}{\alpha_d^2 N}\right)\left(\frac{\alpha_\tau^2 N + 1}{\rho T} + \frac{1-\theta}{M_1}\alpha_\tau^2 N\right). \end{aligned} \tag{30}$$

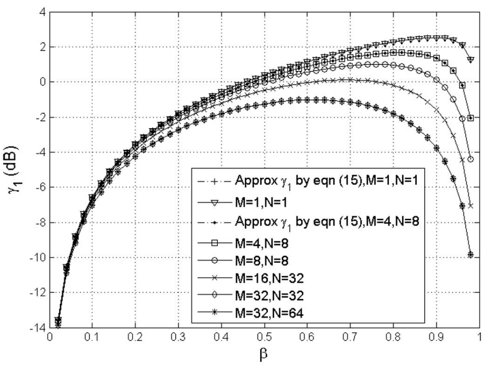

Fig. 1. Effect of number of antennas to optimal β.

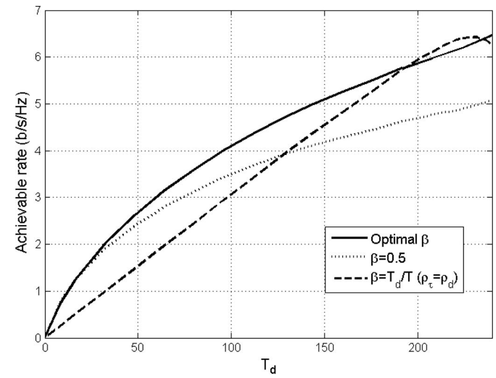

Fig. 2. Achievable rate as a function of T_d with different choice of β.

The parameter $\theta \in (0, 1)$ represents the power imbalance between the two sources, which may be related to the location of the relay and it is assumed fixed.

Our optimization problem is

$$\beta^* = \arg\max_{\beta} \min\{\bar{\gamma}_1, \bar{\gamma}_2\}. \tag{31}$$

The optimal β that solves (31) can be expressed in terms of

$$\beta_i^* = \frac{-b_{3i} + \sqrt{b_{3i}^2 + a_{3i}b_{3i}}}{a_{3i}}, \tag{32}$$

which is the maximizer of $\bar{\gamma}_i$ individually for $i = 1, 2$. Whether β_1^* or β_2^* in (32) solves (31) depends on θ as shown in the following:

Proposition 3: Define $f(\beta) = \bar{\gamma}_1 - \bar{\gamma}_2$. When $M_1 = M_2$, if $\theta > 0.5, f(\beta)$ is a concave function of β and $f(\beta) > 0$ for $\beta \in (0, 1)$. Thus $\min\{\bar{\gamma}_1, \bar{\gamma}_2\} = \bar{\gamma}_2$ and $\beta^* = \beta_2^*$. If $\theta < 0.5, f(\beta)$ is a convex function of β and $f(\beta) < 0$ for $\beta \in (0, 1)$. Thus $\min\{\bar{\gamma}_1, \bar{\gamma}_2\} = \bar{\gamma}_1$ and $\beta^* = \beta_1^*$.

Proof: We have $a_{31}\beta + b_{31} > 0$ and $a_{32}\beta + b_{32} > 0$ for $\beta \in (0, 1)$ and $a_{31} + b_{31} = a_{32} + b_{32}$. Taking the second order derivative of $f(\beta)$ and after some manipulations, we have

$$f''(\beta) = 2\left(\frac{\rho T N \alpha_\tau^2}{M_1 T_d}\right)(a_{31} + b_{31}) \times \frac{b_{32}(1-\theta)^2(a_{31}\beta + b_{31})^3 - b_{31}\theta^2(a_{32}\beta + b_{32})^3}{(a_{31}\beta + b_{31})^3(a_{32}\beta + b_{32})^3}. \tag{33}$$

All the parts in (33) are positive except the numerator of the fraction. Substituting (30) into the numerator and applying the difference of cubes formula on it, its sign is determined by the factor $1 - 2\theta$. When $\theta > 0.5$, which means $\mathbb{S}_2$ has larger power, then $f''(\beta)$ is negative and $f(\beta)$ is a concave function for $\beta \in (0, 1)$. Moreover, we have $f(0) = 0$ and $f(1) = 0$. Thus $f(\beta) > 0$ in $(0, 1)$, implying $\bar{\gamma}_1 > \bar{\gamma}_2$. If $\theta < 0.5$, with a similar argument, $f(\beta)$ is a convex function and $f(\beta) < 0$ for $\beta \in (0, 1)$, implying $\bar{\gamma}_2 > \bar{\gamma}_1$. The optimal β^* is (32) with $i = 1$ if $\theta < 0.5$, and $i = 2$ if $\theta > 0.5$.

IV. Numerical Results and Conclusion

To validate the effectiveness of our method, some numerical results are summarized here, with $T = 256$ and $\rho = 10$ dB. Fig. 1 shows $\bar{\gamma}_1$ versus β with various number of antennas in the symmetric case, with $T_d = 192$. We also illustrate the approximation of $\bar{\gamma}_1$ calculated by (16) for $(M, N) = (1, 1)$ and $(4, 8)$. The results show that the $\bar{\gamma}_1$ through Monte Carlo simulation almost overlaps with the approximation, which demonstrates the correctness of our method. In addition, according to Fig. 1, more energy should be allocated to the training phase to get optimal system performance when the number of antennas increases, which agrees with the inference from (20).

Fig. 2 shows the achievable rate with respect to T_d for the symmetric case, using several representative values of β. Here we set $M = N = 8$. When the optimal β is used, the rate is a monotonically increasing function of T_d and reaches its maximum value at $T_d = T - 2M$. We also simulate the rate for fixed $\beta = 0.5$ and for $\beta = T_d/T$ in which case $\rho_\tau = \rho_d$ always holds. The results for these two cases achieve inferior performance compared to the optimal β, which directly verifies its optimality. Thus we choose T_d as large as possible for better performance.

In conclusion, we propose a power allocation method in the presence of channel estimation in MIMO TWR. We optimize the ratio of training-versus-data for both the symmetric and asymmetric cases. In the symmetric case, with $M_1 = M_2$ and $\theta = 0.5$, the optimal β can be found by solving a fourth order equation, which is further reduced to a quadratic equation when the number of antennas at the sources grows large. Data time is set to its maximum value $T_d = T - 2M$ since the achievable rate is a monotonically increasing function of T_d. In the asymmetric case, we show that the difference of two average SNRs is a concave or convex function for $\beta \in (0, 1)$, depending on θ, enabling the maximization of the minimum of $\bar{\gamma}_1$ and $\bar{\gamma}_2$.

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