

Homeomorphic architecture: Radial prisons and contracted graphs

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Abstract

This paper introduces a type of graph called ‘homeomorphically irreducible tree’ (HIT) and explores its analytical and computational aspects in the architecture of radial prison plans. As a theoretical introduction, HITs are first diagrammatically presented using a taxonomy of 20 different radial prisons. Using this analysis, a generative algorithm that transforms plan connectivity to a simple sequential numeric representation is developed. This method is applied as an architectural plan generator that is parametrically explored using graphs as building skeletons with configurable wing typologies. The aim of the paper is to lay the foundation of a new graph-based approach for the morphogenetic study of symmetry in architectural plans.

Keywords

Homeomorphism, graph, theory, symmetry, morphology

Introduction

Architects, researchers and historians have long sought to find alternative ways of thinking about building forms that often lead to interpretative, comparative or more recently, diverse computational methods. In 20th century, three main computational fields have emerged that deal with both the *qualitative*, performative aspects of various building components and *quantitative*, geometric evaluation of spatial and formal organization of buildings. Within this domain, *space syntax* looks into the patterned and quantitative spatial arrangement of spaces to discover interrelationships and configurations of parts of buildings (Hillier, 1996; Ostwald, 2011; Peponis et al., 1998). This syntactic research investigates alternative ways of capturing the spatial configuration through movement and visibility that can provide insight into the social and cultural functions of space (Peponis et al., 1998). The second field, *shape*

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grammars, offers generative rules that can transform, substitute or reproduce shapes to describe algorithms for a formalist language (Stiny, 1980). In recent years, shape computation has shown major development that can automate the production of emergent sub-shapes and parametric rules (Grasl and Economou, 2013). The third field, *architectural morphology*, aims to formulate abstract models that explore the spatial, formal and performative aspects of buildings (March et al., 1974; Steadman, 1983). In this scientific approach, researchers often formulate ways to analyze the geometric properties of certain building types to provide comparisons and evaluations of various criteria such as circulation, daylight intake or spatial arrangement of rooms. In both cases, the carried analytical research formulates various descriptive models often introducing abstract concepts and mathematical models that can provide novel ways of rethinking about architectural form (Ostwald and Dawes, 2013). These models are then tested on existing building stock, often selected from various historical or modern examples, to prove the performance or relevance of the proposed abstract models (Stiny and Mitchell, 1978). If the transferred knowledge is successful, then the developed computational models can provide novel theoretical insights into certain building typologies or offer simplified mathematical models for the geometric analysis of existing buildings. This aspect of architectural morphology requires a concentrated effort combining different fields such as computation, biology, mathematics, history and theory, while defining its operational domain as a potential bridge between science and the arts (March, 2010; Steadman, 1983).

In pursuit of contributing to the ongoing research on computational morphology, this paper will introduce a class of graphs called ‘homeomorphically irreducible trees’ (HITs) that are applicable to the geometric study of radial prison plans (Gerlach, 2017). As a common test case in syntactical and morphological analyses, radial prisons exhibit diverse symmetrical configurations of similar wing structures that can be represented using graphs. While graph theory has been extensively used in architectural research to look into the adjacency structures of spaces (March and Earl, 1977), plan circulation among rooms (Steadman, 1983) and connectivity of rooms (Ostwald, 2011), homeomorphic trees offer an alternative perspective to the study of architectural morphology by offering simplified numeric representation for branching architectural morphology. Rather than marking individual rooms as nodes of a network (March and Earl, 1977), or describing maximal lines for various shape operations (Grasl and Economou, 2013), homeomorphic trees outline the skeleton of a building form by connecting the local symmetry axis as a directed graph. This principle is introduced by diagramming some of the historical prison plans from different centuries to develop an alternative perspective for the use of contracted graphs to study relationships of symmetry in architectural plan morphology.

Homeomorphically irreducible trees

First discovered by Euler (1707–1782) for the famous Königsberg Bridge Problem in 1736, graph theory has been an exciting field to define geometric problems in applied mathematics (Ostwald, 2011). In general definition, a *graph* is a collection of vertices connected with lines. A subgroup of graphs is called a *tree* that is defined as a connected *acyclic graph*, with no loops occurring among the nodes. This class has found many applications in diverse fields, including computer science, physics, chemistry, engineering, architecture, sociology, genetics and linguistics due to its simple diagrammatic representation and aesthetic appeal (Harary, 1994). A subgroup of acyclic graphs is the HITs that represent unique connectivity graphs described by points and lines (Gerlach, 2017; Haran, 2013). These trees have certain topological properties that determine the complexity of emerging node networks. Firstly,

these trees are *acyclic* where none of the connectivity could lead back to the same node. For instance, a triangle emerging within the tree network is forbidden as it would create a loop (Figure 1). Secondly, the property of ‘homeomorphism’ classifies the emerging trees under isomorphism in conventional graph theory, where a *graph remains invariant to any topological transformation* such as reflection, rotation or scaling (Gamelin and Greene, 1999). Thus, the angles and distances between the nodes of a tree do not play any determinate role in the topology of the tree; all that matters is the amount of connectivity or valency for the nodes. Thirdly, the nodes of the graph need to be *irreducible* where each node cannot have only two connections or a degree of two. This condition rules out any possible trees for three vertices ($n = 3$) (Gerlach, 2017; Figure 1).

By their mathematical definition, HITs are characterized by a single parameter—the total number of nodes in the graph. For instance, for $n = 10$, there are only 10 possible different unique trees (Haran, 2013, see Figure 9 in online Supplemental material). However, the topological complexity and connectivity of these trees cannot be captured by the single parameter of the total node count in the tree. To solve this issue, a distinction between internal and external nodes is proposed (Gerlach, 2017). An *internal* node of a graph has at least three connections. An *external* node only has one connection which is to an internal node. While internal nodes can connect to either another internal or external node, an external node only connects to an internal node because connecting to other external nodes will create cycles violating the first rule mentioned above. This topological property of the HIT offers a numeric representation that can be defined as ‘a collection of partitioned components each centred on their respective internal node’ provided by the Suite of Analytics Software (SAS) solution (Gerlach, 2017). With this model, an HIT can be represented with the degree or valence of internal nodes as a sequence in the convention of $\{i_1, i_2, i_3, \dots, i_j\}$. Under the SAS model, the unique trees for $n = 10$ are redefined in the format of $\{9\}, \{3,3,3,3\}, \{3,3,3,3\}, \{4,6\}, \{5,5\}, \{4,3,4\}, \{3,5,3\}, \{3,4,4\}, \{3,7\}, \{3,3,5\}$, respectively, all of which have a total of 10 nodes. This way the numeric representation not only captures the amount of internal and external connections and propagation of the network but also can say a lot about the overall architecture of the graph using simple sequential numeric parameters.

The emerging graphs show that HITs try to display a state of equilibrium among node configuration while distributing radial symmetries along an axis according to the number of connections to internal nodes. However, the visual representation of constant degree value HITs presents some special cases. For instance, the tree $\{3,3,3,3\}$ for $n = 10$ can be represented in two different ways since the SAS model signifies neither the identity of the internal

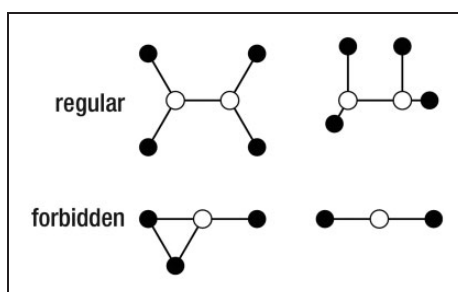


Figure 1. Regular and forbidden cases of homeomorphism. Top row shows two homeomorphically identical trees with six total nodes. Bottom row shows forbidden cases producing a loop (left) and a reducible two-legged node (right).

connectivity nor the direction of connections. Furthermore, representing different topological complexities with single numbers lacks a hierarchical structure. One benefit of HITs is visible for the non-constant degree values that are often visualized axially where external nodes are distributed radially to show a balanced positioning of parts. In these trees, the notational scheme becomes more intuitive where the linear progression of the connected internal nodes represents the overall topology of the tree. This property also renders notation for identical HITs such as {4,6}, {6,4} as equivalent, and signifying both of them becomes redundant. Although the SAS model has limitations to represent radial trees as starting from a central node lacks directionality, it offers a robust model for axial HITs that can be numerically reduced to a sequence.

Homeomorphic architecture

In architecture, graph theory has become a major tool to study spatial distribution, configuration, adjacency and relationship of rooms within building plans (March, 2010; Ostwald, 2011; Steadman, 1983). In these examples, the spaces are often visualized as trees that reveal their connectivity within a network of nodes. Although architecture does not often follow abstract and implicit mathematical formulas, homeomorphism presents a growth mechanism that is applicable to the study of building plans that exhibit symmetry in their formal configuration. Among many historical buildings that can be analyzed, these characteristics are mostly observed among hospitals, asylums, prisons, military barracks and schools that exhibit patterned wing development and repetitive spatial division (Steadman, 2014; Vessella, 2017). In institutional typologies, plans are mainly organized with repetitive rooms distributed along doubly loaded corridors with centralized circulation cores, where the overall building form branches out towards an open landscape. Due to their large footprints, buildings like pavilion hospitals, asylums and prisons are often placed in suburban areas making their integration into urban environments difficult while their programs are distributed into clustered wings that describe an HIT network.

Before exploring the computational aspects of homeomorphism, the following section will first investigate its morphological characteristics by analyzing some of the historical building types. In order to make the arguments comprehensive and morphological analysis as consistent as possible, homeomorphism is presented using radial prison plans spanning different centuries that show different branching morphologies. To determine the HIT graphs of the plans, a process akin to ‘skeletonization’ is used where wing and corridor terminations are marked with external nodes and intersections of corridors are marked with internal nodes (Lakshmi and Punithavalli, 2009). These nodes are then connected to other internal axis and additional convergence points that in most cases directly overlap with the internal circulation network or symmetry axis of wings or buildings. In some cases, the corridors can also terminate into a room; however, this does not change the axial representation of the HIT graph. A general property of the HIT graphs is the overlap with the internal circulation of plans that can be characterized by a double-loaded corridor while offering a novel and simple way to trace the symmetry of the morphological skeleton of buildings using its formal boundary.

Prison morphology

One of the building typologies that exhibits HIT properties is the radial prison that emerged in the late 18th century. A pioneer in the design and construction of this typology was Jeremy Bentham who invented the ‘panopticon’ model in 1787—a radial plan that allowed

all prisoners to be observed with a single watchman located at the centre (Bentham, 1791). As the growing number of inmates steadily rose throughout the 19th century, panopticon prisons turned out to be extremely costly causing the state officials and taxpayers to question the appropriateness of the structures that were built. The radial design also had various functional flaws that did not provide stealth and privacy for the guards since both the prisoners and guards could see each other (Steadman, 2014). Later, Bentham's utilitarian designs were transformed into replicable and low-cost modern structures. These were aimed at restricting and controlling the movement of prisoners throughout the facility in segmented modular wings that also required smaller prison staff to monitor prisoners directly. Furthermore, the form of the prisons transformed into radiating structures where their large footprints and clustered arrangement required extensive open lands that restricted their integration within the urban environments (Johnston, 2006).

Historically, prison architecture evolved through different types of architectural plans and programmatic specifications. In the USA, two of the earliest advancements after the panopticon were the Philadelphia and the Auburn model. In the former, multiple radiating wings were connected at centres that provided ease of administration; whereas in the latter, the radical programmatic division brought daytime association via devoted outdoor labour in complete silence followed by nighttime separation in isolated cells (Vessella, 2017). These models were later amalgamated in the 'telephone-pole' model that connected separated structures along a spine providing accessibility (Steadman, 2014; Vessella, 2017). An early example of this type was the Fresnes Prison built near Paris in 1898 that influenced many subsequent designs that are still used to this day. Apart from these models, in 20th century, new prison models with courtyards and detached structures were created that tried to solve some of the ventilation and daylight problems of cells. With the use of technological surveillance systems, centralized and closed systems were gradually replaced with more open layouts. In contrast to the earlier branching plans of the radial prisons that provided closed circuits for optimized administration and internal circulation of inmates, the modern prison gave more formal freedom to the designers who could operate on larger suburban landscapes with different building types in telephone-pole or distributed layouts (Fransson et al., 2018; Vessella, 2017).

Among the historical prison types, homeomorphic properties are mostly found in the highly centralized and closed structure of the radial prisons that were influential during the 19th century. Two early examples of this type were the Eastern State Penitentiary in Philadelphia and Moabit prison in Berlin (Figure 2) that expanded the doughnut-shaped plans of panopticons into more tree-like cluster networks (Steadman, 2014). In the former, the circular plan scheme was transformed into a radiating one, satisfying the requirements of confinement and control of inmates while increasing the outer surface area of the prison walls to place more cells with access to air and light (Johnston, 2006; Norman, 1964). These prisons were characterized by the modular distribution of similar wing structures around centralized internal nodes that turned the plan into a tree. Most of the internal circulation was in the form of a doubly loaded corridor that can also contain vertical galleries, bridges and skylights with multiple floors of cells contained in wings (Steadman, 2014). With this approach, each wing is capable of achieving different lengths and number of cells making the branching morphology of the plan as the primary source for an HIT organization.

Radial prison taxonomy

To visualize the presence and variations of the homeomorphic wing architecture, an array of radial prisons spanning different centuries and geographies are chosen. In Figure 3, a

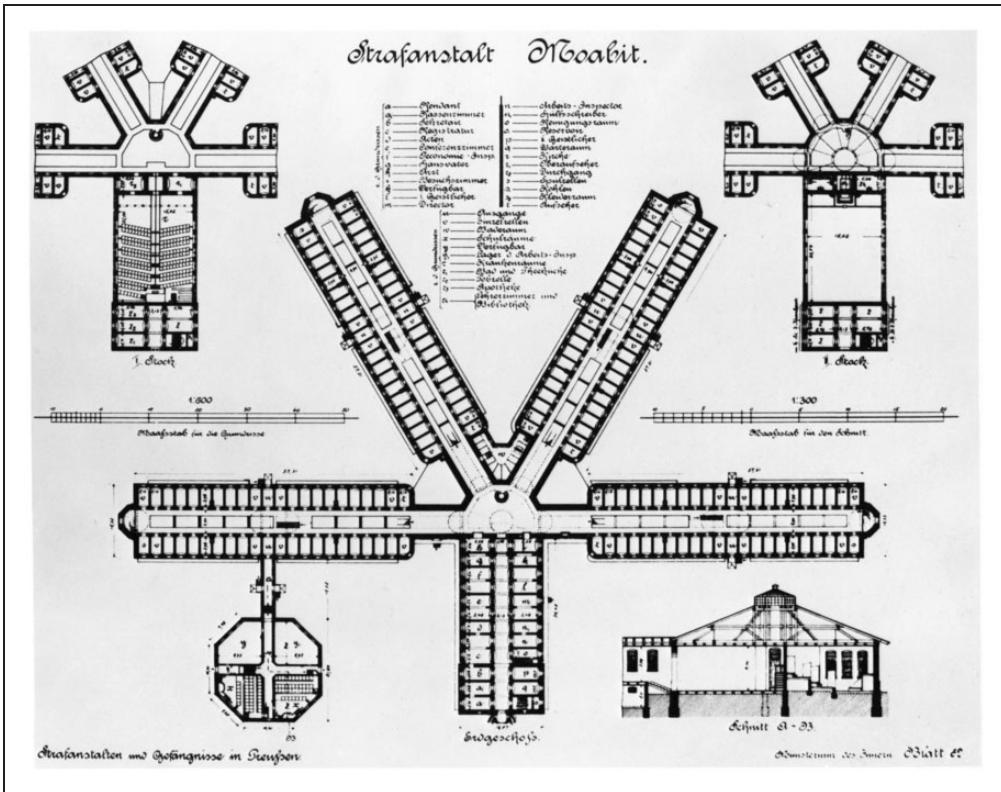


Figure 2. Moabit Prison in Berlin showing an example of the “radial prison” plan. Source: From Karl Krohne and Uber R, *Die Strafanstalten und Gefängnisse in Preussen*, Berlin Carl Heymann, 1901, atlas plates 50–52.

taxonomy of 20 different samples is presented with various sizes and number of wings with double-loaded corridors to study the morphological characteristics of the radial prison archetype. An observation into the plans reveals two main characteristics for the radial prisons. Firstly, prisons like Eastern State Penitentiary, New Jersey and Wandsworth display a case of growth over time where multiple wings can branch out towards the landscape or can be attached or symmetrically aligned to pre-existing structures to increase capacity. Secondly, radial prisons predominantly display morphological isomorphism for wings that are connected at centralized cores for surveillance. In Kresty, Singapore, Leuven, New Jersey and Pentonville, all the wings contain similar cellular division, whereas in Singapore, Saint-Paul, Stuttgart and Ratibor there are variations among wing plans. In some cases, the radial prison is connected to a separate entrance structure that exhibits different forms, while often aligning with the symmetrical organization of the prison (Leuven, Wandsworth, Pentonville, Mazas, ESP, Vridsløselille). Most of the radial prisons are located in rectangular or polygonal plots making their divergent structures predominantly organized according to intrinsic mathematical principles that lack a direct input from site conditions. This homeomorphic structure becomes more evident when internal corridors of the structures are traced as a graph to visualize the formal organization of the building plans as a skeleton (see Figure 10 in SM). For these diagrams, a root node is defined that either overlaps with a radial centre where multiple wings branch out or represent an

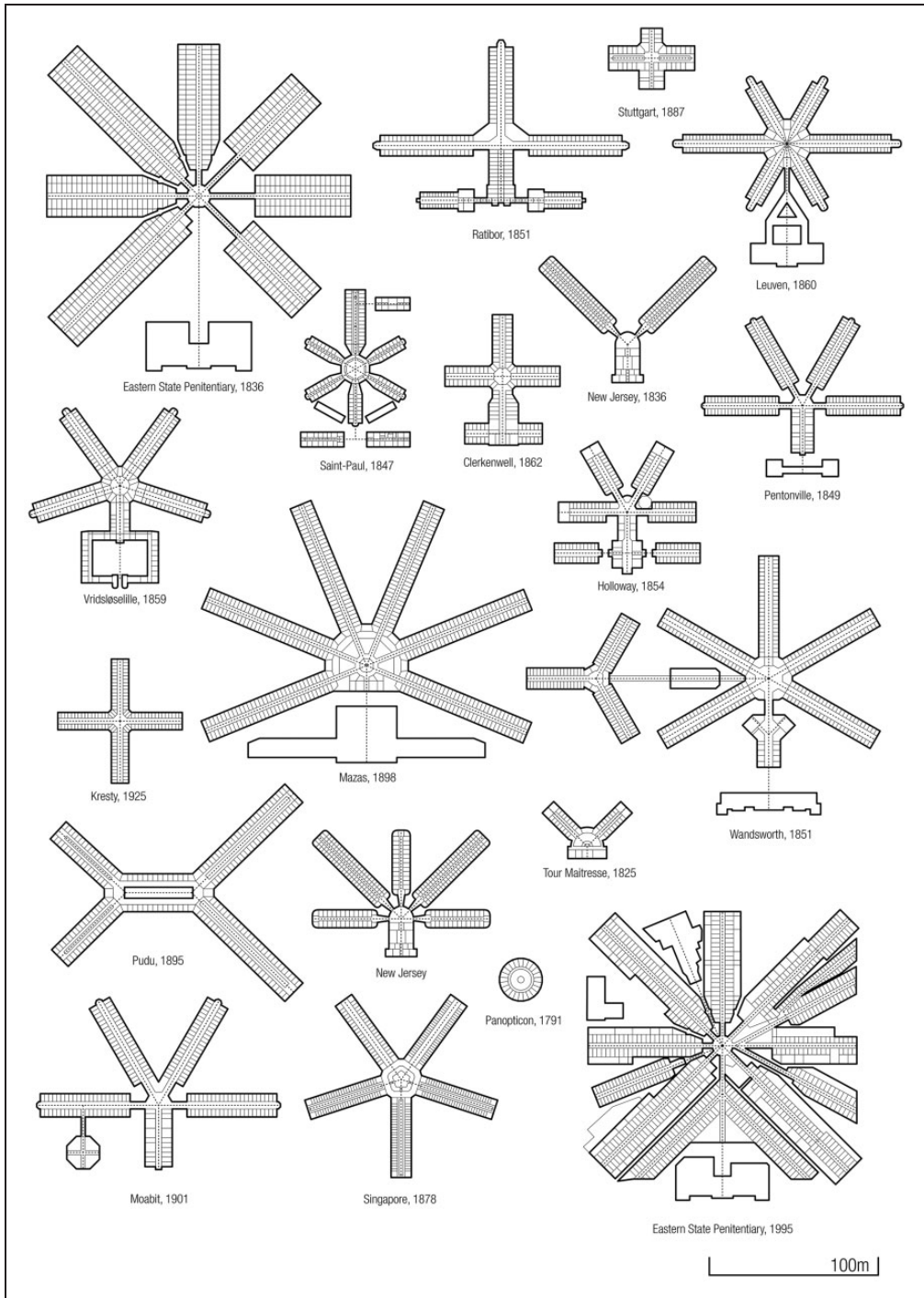


Figure 3. A taxonomy of radial prison plans showing different branching morphologies and networking structures.

additional entrance structure where prison wings are connected as a directed graph with radiating and bifurcating segments. With this depiction, most of the morphological complexity can be simplified and the similarities of symmetry among radial prisons captured. These diagrams show that radial prison typology mostly exhibits homeomorphism among the internal connectivity of plans that can be parametrically varied, while radial configuration of wings often shows isomorphism.

Computation of homeomorphism

Architectural plans can present a lot of formal complexities that make them challenging to determine the organizing principles behind the geometrical distribution of elements. With current digital technologies, architects can choose between developing free-form buildings that may follow various parametric workflows, or could arrange buildings through orthogonal or polygonal geometries following natural packing principles for internal spaces (Steadman, 2006). While computation or classification of the free-form buildings remains a challenging issue due to the amount of formal asymmetry, it is plausible to follow an alternative route, even an opposite agenda, to look for mathematical structures for the study of architectural symmetry prior to tackling issues of asymmetry in architectural form. This perspective stems from the lack of formal analysis or development on the notion of symmetry in architecture that remains as a computational, theoretical and historical problem (Lynn et al., 1995) while the term has diverse theoretical and practical applications in other natural sciences (Gardner, 1990; Wely, 2005). In this regard, homeomorphism offers an alternative formal computation for architectural morphology where building plans can be generated directly from the symmetry axis. Compared to shape grammars that consider symmetry in relation to the orientation, reproduction or modification of modules (Grasl and Economou, 2013; Stiny and Mitchell, 1978), *morphogenesis* begins with the description of abstract mathematical models and generates a form using simple geometric operations that increase definition and differentiation of form progressively (Hensel et al., 2006). This approach has the potential to offer an organic perspective to architectural morphology where *additive* (parts to whole) and *subtractive* (whole to parts) methods can be combined under rules of symmetry and proportion.

Computation of homeomorphic plan form

In many historical radial prison plans, the symmetry axis coincides with the double-loaded circulation connecting internal spaces or rooms within wings. This tree-like skeleton of the plan particularly expresses an inside-out growing boundary that determines the overall form of buildings. In the following model, this emphasis is transformed into a formal algorithm to consider symmetry in relation to proportion while issues of scale and measurement in architecture are ignored to visualize the potential computational applications and simplicity of the homeomorphic graph approach. To generate the plans, a two-step protocol is established akin to organic growth in natural forms. In the first step that is *additive*, the axial organization of the plan grows outwards by recursively adding new nodes to a tree and defining the overall form boundary of the plan by offsetting the emerging graph. In the second step, the formal boundary defined by wings and cores is subdivided internally to place rooms or cells. This *subtractive* step is considered due to the rectangular packing and size similarity of prison cells (Steadman, 2006). This way, morphological computation can be carried as if the plan form is fully grown, where *additive* and *subtractive* methods can be subsequently applied through recursive geometric operations.

To investigate radial prison morphology in a computational manner, a simple algorithm structured around the mathematical notion of homeomorphic trees is developed that can generate radial building plans with internal corridors. This recursive algorithm is developed in Python using the main principles highlighted above, where the formal complexity of a tree graph can be represented with a sequence of numbers and hierarchy among nodes (Gerlach, 2017). In Figure 4, the procedural development of this algorithm is visualized for the HIT of $\{R \rightarrow 3 \rightarrow 3 \rightarrow 4\}$ containing 11 nodes. Starting from a root node (R), the algorithm uses numeric code to propagate the tree structure in a linear fashion producing a major symmetry axis. Each number in the sequence marks the number of external nodes to be connected to a preceding internal or root node. For simplicity, the nodes overlapping with the horizontal axis are chosen as the internal nodes for subsequent branches. At each step, external nodes are connected to internal nodes in a uniform fashion producing equal angles between the branches defining the recursive additive process. To define the form of the prison, the emerging skeleton is offset in three steps. The first offset determines the space for internal circulation by perpendicularly offsetting graph axis. At interior nodes, connectivity information is used to define bisector lines to find offset directions. The second offset occurs along the axis connecting to external nodes to define the spaces for wings (Figure 5). The third offset occurs in a similar fashion around internal nodes using bisector lines. The cores and wings then undergo separate topological division to achieve the spatial distribution of cells and spaces. Using these HIT rules, permutations of various architectural plans using the existing mathematical codes for 4, 5, 6, 7, 8, 9 and 10 node trees are generated to visualize the morphogenetic potential of the approach (see Figure 10 in SM).

Expansion and subdivision protocol of wings and cores

In the second part of the algorithm, emerging wing and core boundaries undergo separate subdivision protocol that follows similar geometric principles and parameters. Each wing is composed of two symmetrical halves that are defined by an internal corridor and external-

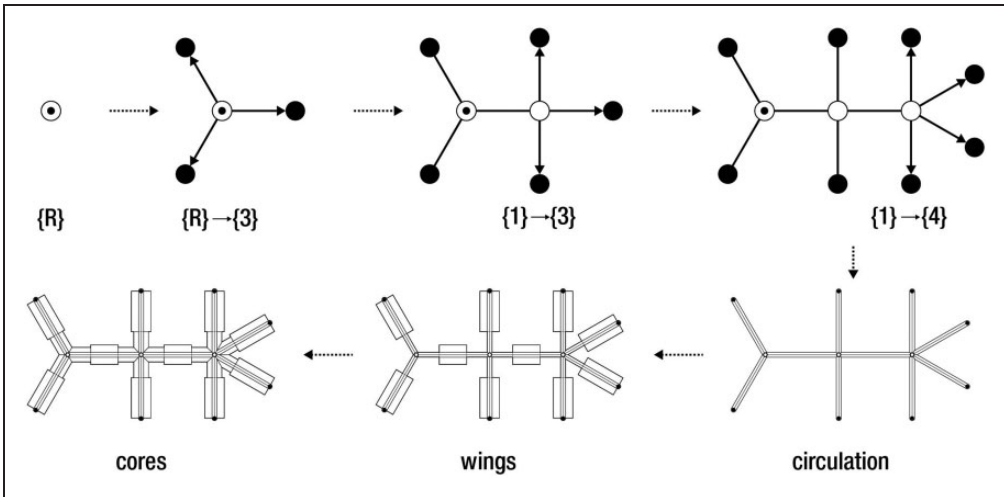


Figure 4. Generative modelling protocol for $\{R \rightarrow 3 \rightarrow 3 \rightarrow 4\}$. Internal nodes are marked with white, and external nodes are marked with black dots. Each step shows the addition of external branches to achieve the total number of connections to an internal node. Each axis is then thickened first into the inner circulation corridors before wings and cores are further expanded.

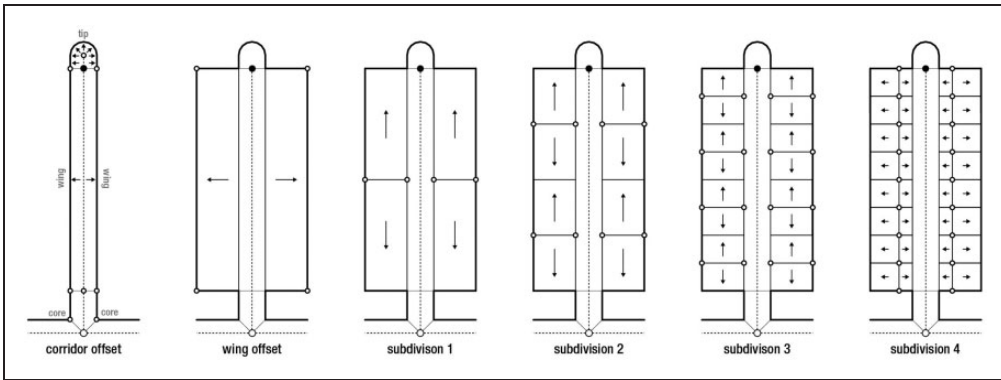


Figure 5. Expansion and subdivision protocol of prison wings.

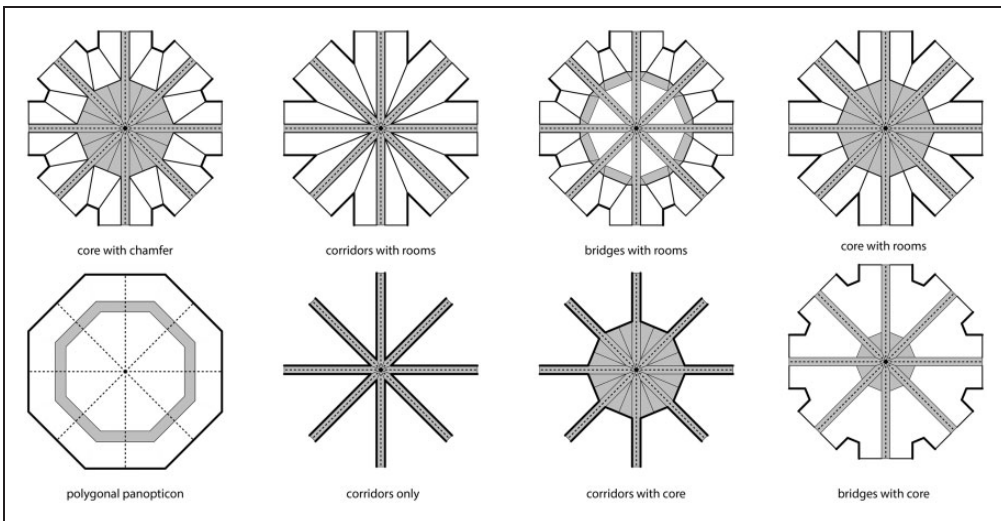


Figure 6. Morphological comparison of various prison cores.

facing spaces (Figure 5). For the latter, each emerging rectangular polygon can be recursively subdivided into two halves using a division threshold, progressively producing smaller spaces or rooms. Additional topological characteristics of the prison wings are added around this subdivision protocol that can accommodate variations. These include attachment/detachment of wings along the symmetry axis, bridge connections to internal nodes, tapered bases, rooms located at the end of an axis, rounded tip expansions and gallery spaces located at corridors (Figure 12 in SM). All of these topological variations are configured with binary parameters that give the option of 128 configurations for wing plans (for permutations of wings, see Figure 14 in SM).

For the cores, there are two possible formal propagations that can produce an expanded radial prison or a polygonal panopticon. In radial prison internal nodes, the first two offsets of the axis follow the wing protocol, where the internal circulation corridors and the external formal boundary are established that can also be chamfered diagonally (Figure 13 in SM).

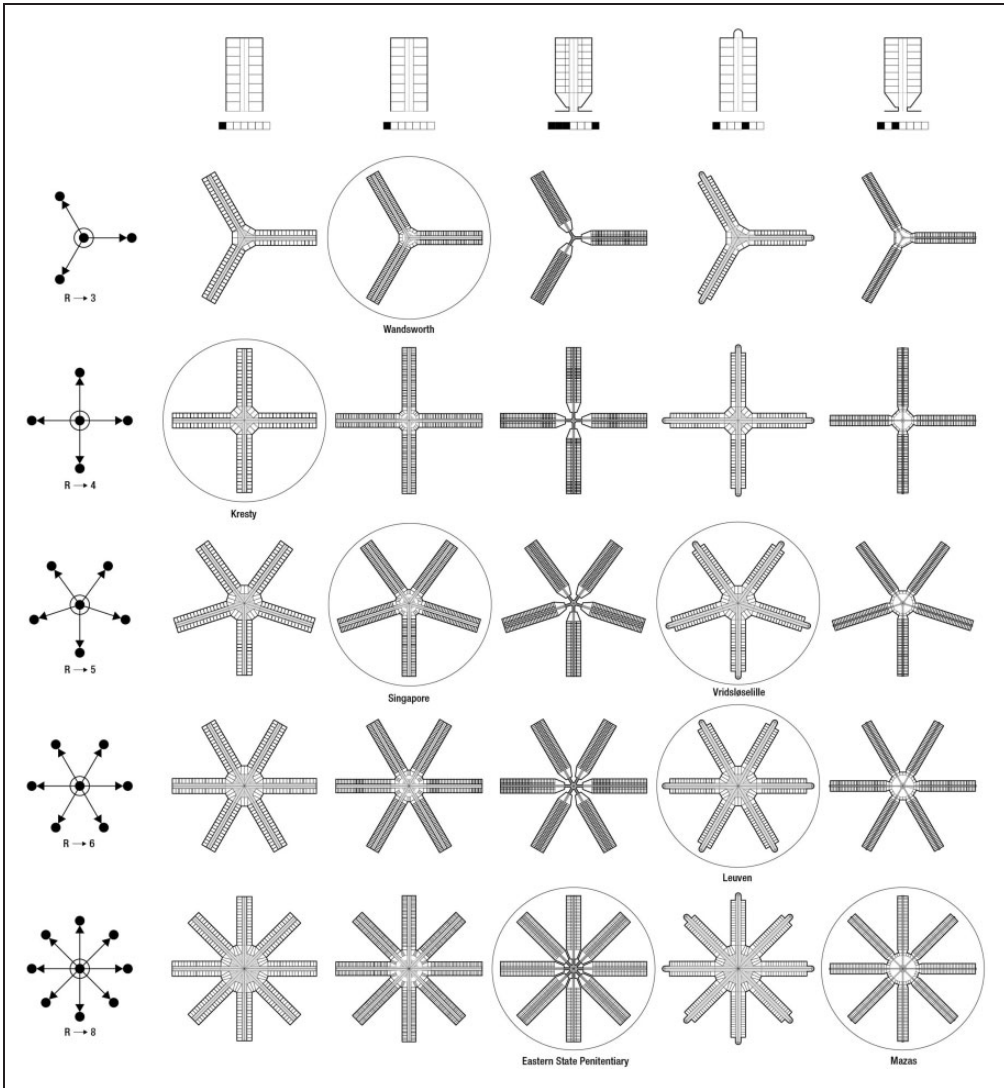


Figure 7. Morphological matrix for radial prisons with single number homeomorphism. Left column shows HIT diagram of plans. Top row shows wing configuration.

In panopticons, there is no initial offset for the corridor; instead, the form offset is brought to the maximum to achieve a closed polygonal boundary. In the first subdivision, the emerging spaces are retreated to gain more space for circulation (radial) or gallery (panopticon). A second subdivision defines an additional corridor space to access the core rooms. With these similar rules, closed panopticon types and open radial types can be morphologically related as both of them can be generated from a centre using polygonal offsets (Steadman, 2014). In Figure 6, several configurational variations for radial prison cores are shown that can also be found among the analyzed samples in taxonomy (Figure 3).

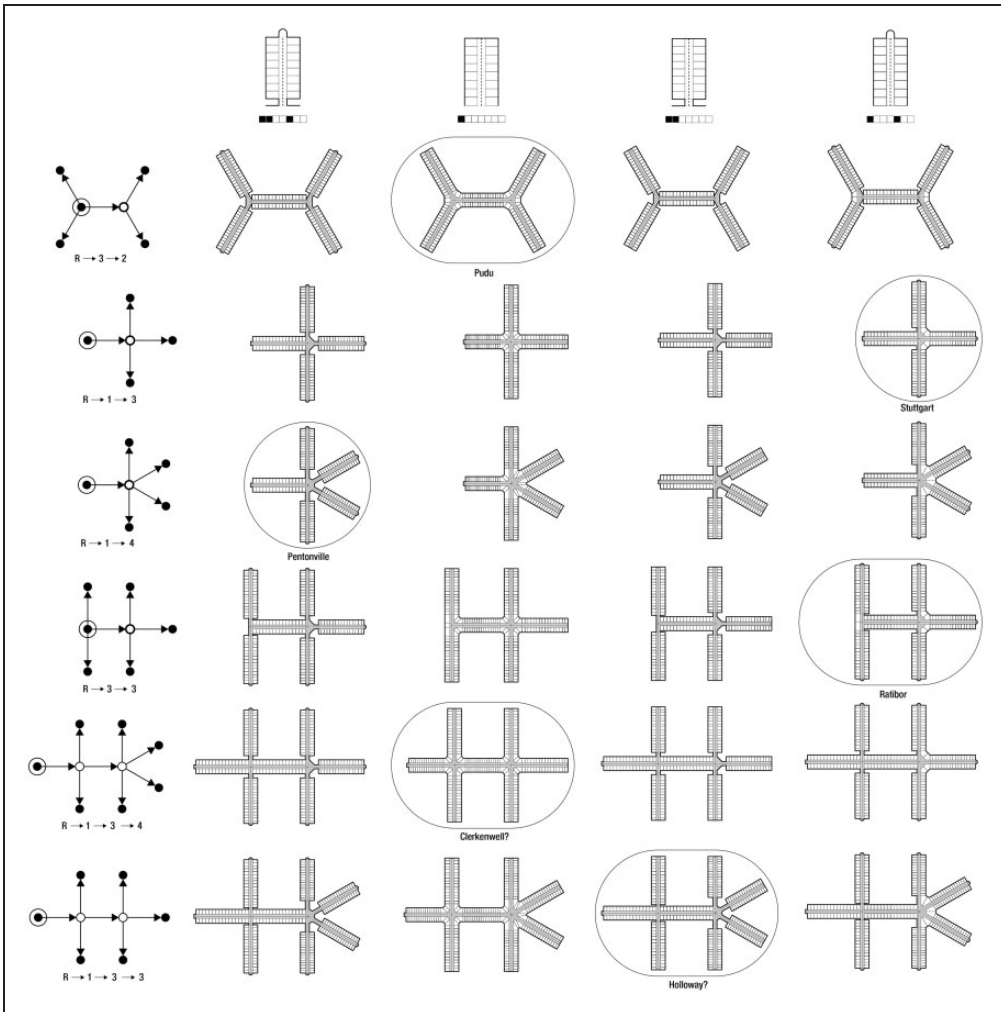


Figure 8. Morphological matrix for radial prisons with multiple numbers of homeomorphic trees. The left column shows HIT diagram of plans. The top row shows wing configuration.

Parametric computation of radial prisons

In the next phase, the archetypal characteristics and morphogenetic variations of the radial prison are explored through various homeomorphic configurations (Figures 7 and 8). In these samples, the wing types and topological parameters are kept identical among trees with different node counts. Furthermore, the wings are shown as fully grown with variation types extracted from permutations in Figure 14. Among the single number trees radiating from a central core R , external nodes of 3, 4, 5, 6 and 8 are plotted for five different wing configurations (Figure 7). In this configuration matrix, seven of the radial prisons are found with three-legged Wandsworth and five-legged Singapore; five-legged Vridsløselille and six-legged Leuven show morphological similarities. Among the trees with $R \rightarrow 8$, Eastern State Penitentiary and Mazas are shown with full-grown versions. Other radial variations are also displayed to show the generative capacity of the approach.

For homeomorphic plans with multiple numbers of internal nodes, six different trees are investigated (Figure 8). Among these, Pudu is found in $R \rightarrow 3 \rightarrow 2$ with chamfered cores and Pentonville is found in $R \rightarrow 1 \rightarrow 4$ with the first branch joining against the core of the radial wings. Apart from these samples, plans that show kinship to Ratibor, Stuttgart, Clerkenwell and Holloways prisons are found in more complex cases. However, due to the topological variations and lack of identity among their wings, the generated samples do not replicate the plans in taxonomy; instead, the variations display the normalized cases with the similar tree graphs. While these variations conform with the overall morphology of the analyzed radial prison plans, each wing and core require further differentiation to capture more articulate plans. To keep the simplicity of the morphological computations and introduction of the method, these additional rules are not considered in this paper.

Conclusion

This paper introduced a mathematical term ‘homeomorphically irreducible tree’ to offer a simplified graph approach for the study of radial prison plans. The concept is explored from two alternative directions, through historical–analytical and computational–generative studies, to offer a robust study of plan morphology. The results show that HIT algorithm offers potential that can be either used as an analytical tool for a geometrical study of building plans exhibiting symmetry, or as a generative modelling tool that can produce permutations of architectural plans.

The main characteristic of homeomorphic architecture is the sequential numeric code for branching morphologies in the form of $\{R \rightarrow i_1 \rightarrow i_2 \rightarrow i_3, \dots, i_j\}$ combined with various topological parameters for internal and external nodes. This procedural generative model combines *additive* and *subtractive* methods, by taking the linear hierarchical structure of directional graphs and combining it with inner subdivision of emerging forms that can generate wide parametric variability (Harding and Shepherd, 2017). With this model, HITs can capture the morphological skeleton of axial and branching buildings by tracing the configuration of wings. While the analyzed prison plans show homeomorphic properties, further investigation into the directionality and symmetry properties of HIT is required to offer a broader investigation of its morphospace and analysis of other wing-based building types.

The algorithmic simplicity of HIT can be used to carry more in-depth research of other historical examples to develop meta-models that can help determine morphological characteristics and stylistic building codes. Some of the generative studies presented in this paper already overlap with other building typologies and programs such as offices, housing or hospitals. This way, HITs can offer a *simple abstract* model to generate building forms to speculate a new formulation for building archetypes that are structured around the notion of symmetry and growth (March and Steadman, 1974; Steadman, 2014). To conclude, compared to shape grammars, homeomorphic architecture offers an alternative method for the application of graph theory where building plans can be primarily studied through symmetry where inner spatial division can be achieved directly from partitioned branches of a thickened morphological skeleton.

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Supplemental material

Supplemental material for this article is available online.

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