Optical Index-Coded Space Shift Keying (IC/SSK)

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***Abstract*—In this letter, a new channel coding technique is proposed for optical spatial shift keying (SSK)-based visible light communications (VLC) systems by introducing redundancy in the spatial domain. The information is encoded by the redundant indices of the light emitting diodes (LEDs), assuming full channel state information (CSI) at the transmitter. The efficient construction of index-coding (IC) is presented with low computational complexity. Depending on the number of LEDs and the PDs as well as the amount of redundancy, it is concluded that IC/SSK achieves excellent coding gains at full rate and given target bit error rate (BER), as contrasted to the conventional channel coding in bit-domain, and yields superior BER performance in the presence of the VLC *blockage channels* as compared to the uncoded SSK systems.**

***Index Terms*—Optical wireless communications (OWC), space shift keying (SSK), index-coding (IC), blockage channels, visible light communications (VLC).**

I. Introduction

SPACE shift keying (SSK) is a modulation scheme for multiple-input multiple-output (MIMO) channels, based on spatial modulation (SM) concepts to provide better performance over conventional amplitude/phase modulation (APM) techniques. In SSK, a randomly chosen antenna index is used during transmission that relays information, rather than the transmitted symbols themselves [1]. Because of its simplicity and its characteristics, SSK seems to be a proper modulation technique for low complexity implementation of MIMO-based communications. Therefore, it is especially appropriate for optical wireless communications (OWC), which employs incoherent light sources and uses intensity modulation at the transmitter and direct detection at the receiver side [2],[3]. Recently it has been shown that the SSK was also a very effective and important transmission scheme for physical layer security in VLC [4],[5].

On the other hand, conventional channel coding in digital communications improves the error performance of the system a great deal in the presence of additive noise, by encoding the message with redundant information. Inspired by the channel coding, in this letter we present a new channel coding scheme for SSK, called index-coding SSK (IC/SSK) where redundancy is introduced by increasing the number of randomly chosen LEDs greater than one. To design IC/SSK codes close to optimal one, full channel state information (CSI) is required at the transmitter. However, this would pose some practical implementation problems due to a large feedback overhead during transmission of the CSI by feedback possibly infrared channel. In order to reduce the feedback overhead, the universal codebook-based technique is widely adopted, which is shared by the receiver and the transmitter. A pre-designed IC/SSK code is selected from the codebook by the receiver that maximizes the channel capacity and only its label is transmitted back to the transmitter [6]. Once the channel is estimated at the receiver and fed back to the source, the CSI needs to be updated infrequently. Designing an efficient index-based coding then becomes possible at the source to maximize the Euclidean distance among the SSK signal constellation. In [1], the SSK concept was extended to incorporate classical channel coding based on a bit-interleaved coded modulation. But, the spectral efficiency of the system in terms of bits/sec/Hz drops with the coding rate, depending on the encoding employed in the system. However, as will be shown in the following sections, there is no information rate loss in the IC/SSK system, caused by the encoding process since the redundancy in coding is provided in spatial-domain. On the other hand, to improve the spectral efficiency of the SSK system, a generalized SSK (GSSK) scheme was proposed in [7]. In GSSK, as opposed to the SSK, more than one antenna element is chosen randomly at each signaling interval to transmit more bits per second than that of the SSK. Nevertheless, there is an essential difference between the IC/SSK and GSSK in that, the former aims at improving the error performance of the SSK system, whereas the latter is mainly for increasing the spectral efficiency by multiple active transmitting elements, where it also creates a higher transmit diversity.

II. Index Coding in SSK Systems

Unlike conventional SSK, the proposed method takes advantage of the spatial-domain for modulation and coding simultaneously. The encoder at the transmitter introduces some redundancy in terms the number of the randomly chosen active LEDs to cope with channel noise, where it also creates a system with higher transmit diversity. The simple and yet effective combination of IC and SSK is shown to improve the BER performance significantly compared to the conventional SSK-based transmission method. Accordingly, in every signaling interval T_s, a randomly generated bit stream vector $\mathbf{b} = (b_0, b_1, \cdots, b_{\ell-1})$ is represented by a message symbol m, taking value in the N_t-dim Galois Field, GF(N_t), with the elements in $\mathbb{M} = \{0, 1, \cdots, N_t - 1\}$, where $\ell = \log_2(N_t)$. Then a *non-binary* encoder takes the message $m = i \in \mathbb{M}$ and encodes into a codeword $\mathbf{c} = (i_1, i_2, \cdots, i_{n_t})$ of n_t symbols, all taking values in GF(N_t). Subsequently, the codewors are mapped to N_t-dim signal constellation vectors

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TABLE I. Transmission vectors of IC/SSK for $N_\mathrm{t} = 8$ and $n_\mathrm{t} = 3$

$\mathbf{b} = [b_1, b_2, b_3]^\mathrm{T}$	m	$\mathbf{c}_k = (c_{k,1}, c_{k,2}, c_{k,3})$	$\mathbf{s}_k = [s_{k,1} \cdots s_{k,8}]^\mathrm{T}$
[0 0 0]	0	$\mathbf{c}_1 = (\ 0\ 1\ 2\ )$	$\mathbf{s}_1 = [\frac{1}{\sqrt{n_t}}\ \frac{1}{\sqrt{n_t}}\ \frac{1}{\sqrt{n_t}}\ 0\ 0\ 0\ 0\ 0]^\mathrm{T}$
[0 0 1]	1	$\mathbf{c}_2 = (1\ 2\ 3\ )$	$\mathbf{s}_2 = [0\ \frac{1}{\sqrt{n_t}}\ \frac{1}{\sqrt{n_t}}\ \frac{1}{\sqrt{n_t}}\ 0\ 0\ 0\ 0]$
$[0\ 1\ 0]^\mathrm{T}$	2	$\mathbf{c}_3 = (\ 1\ 4\ 5\ )$	$\mathbf{s}_3 = [0\ \frac{1}{\sqrt{n_t}}\ 0\ 0\ \frac{1}{\sqrt{n_t}}\ \frac{1}{\sqrt{n_t}}\ 0\ 0]^\mathrm{T}$
[0 1 1]	3	$\mathbf{c}_4 = (\ 1\ 5\ 6\ )$	$\mathbf{s}_4 = [0\ \frac{1}{\sqrt{n_t}}\ 0\ 0\ 0\ \frac{1}{\sqrt{n_t}}\ \frac{1}{\sqrt{n_t}}\ 0]^\mathrm{T}$
[1 0 0]	4	$\mathbf{c}_5 = (\ 2\ 4\ 5\ )$	$\mathbf{s}_5 = [0\ 0\ \frac{1}{\sqrt{n_t}}\ 0\ \frac{1}{\sqrt{n_t}}\ \frac{1}{\sqrt{n_t}}\ 0\ 0]^\mathrm{T}$
[1 0 1]	5	$\mathbf{c}_6 = (\ 3\ 4\ 5\ )$	$\mathbf{s}_6 = [0\ 0\ 0\ \frac{1}{\sqrt{n_t}}\ \frac{1}{\sqrt{n_t}}\ \frac{1}{\sqrt{n_t}}\ 0\ 0]^\mathrm{T}$
[1 1 0]	6	$\mathbf{c}_7 = (\ 3\ 5\ 6\ )$	$\mathbf{s}_7 = [0\ 0\ 0\ \frac{1}{\sqrt{n_t}}\ 0\ \frac{1}{\sqrt{n_t}}\ \frac{1}{\sqrt{n_t}}\ 0]^\mathrm{T}$
[1 1 1]	7	$\mathbf{c}_8 = (\ 5\ 6\ 7\ )$	$\mathbf{s}_8 = [0\ 0\ 0\ 0\ 0\ \frac{1}{\sqrt{n_t}}\ \frac{1}{\sqrt{n_t}}\ \frac{1}{\sqrt{n_t}}]^\mathrm{T}$

$\mathbf{s} = \left[0, 1/\sqrt{n_t}, 0, \cdots, 1/\sqrt{n_t}, \cdots, 0, 1/\sqrt{n_t}\right]^T$. The locations of the nonzero elements of $\mathbf{s}$ are $i_1, i_2, \cdots, i_{n_t} \in \mathbb{M}$. Hence, the *one-to-one* mapper jointly maps and encodes the input bits into IC/SSK transmission symbols set as follows:

$$\mathbf{b} \in \mathbb{B} \ \Leftrightarrow\ m \in \mathbb{M} \ \Leftrightarrow\ \mathbf{c} \in \mathbb{C} \ \Leftrightarrow\ \mathbf{s} \in \mathbb{S}.$$

Note that one of the main advantages of the index coding over the conventional binary coding is that the source information rate R_b bits/sec of binary coding drops to $R_c = (k/n)R_b$, where k and n are bit-lengths of the uncoded and coded words, respectively, while R_b would not be affected by the index coding mainly because coding is implemented in the spatial-domain.

As an example, transmitting part of an IC/SSK system with $N_t = 8$, and $n_t = 3$ is shown in Table I, where the coded symbols $\mathbf{s}_k$ with n_t repetitions are chosen to make the average transmit signal unity (i.e. $\mathrm{E}\{\mathbf{s}_k^H \mathbf{s}_k\} = 1,\ \forall k$) as each nonzero component of $\mathbf{s}_k$ takes value $1/\sqrt{n_\mathrm{t}}$. Thus, the transmit signals in IC/SSK can be formulated as

$$\mathbb{S} = \left\{ s_{k,i} \in \{0, \frac{1}{\sqrt{n_t}}\} \;\middle|\; \sum_{i=1}^{N_\mathrm{t}} s_{k,i} = \sqrt{n_t},\ \text{for}\ \ 1 \le k \le N_t \right\}. \quad (1)$$

It is important to note that choosing an optimal code $\mathbb{C}$, with N_t codewords, out of $G = \binom{\binom{N_\mathrm{t}}{n_\mathrm{t}}}{N_\mathrm{t}}$ possible code set combinations has a significant impact on the bit error ratio (BER) performance of the IC/SSK systems. When the channel state information (CSI) is available at the transmitter, it is possible to select the optimal code maximizing the minimum Euclidean distance, $d_{\min}$, among the effective coded constellation symbols $\mathbf{u}_{k,\mathrm{eff}} = \mathbf{H}\mathbf{s}_k$. However, this optimization can be shown to be NP-hard and becomes infeasible for large N_t and n_t values. The construction of a suboptimal code design will be detailed in the following subsection. The IC/SSK modulated symbols are transmitted over $N_\mathrm{r} \times N_\mathrm{t}$ OWC channel $\mathbf{H}$ and $N_\mathrm{r} \times 1$ received baseband electrical domain signal vector is obtained as

$$\mathbf{y} = \mathbf{H}\mathbf{s}_k + \mathbf{w} \quad (2)$$

where additive white Gaussian noise (AWGN) vector $\mathbf{w} \in C^{N_\mathrm{r} \times 1}$ has independent entries according to $\mathcal{N} \sim \left(0, \sigma_w^2\right)$. The analog-to-digital and digital-to-analog conversations are assumed to be unity without loss of generality. Since the vector $\mathbf{s}_k$ in (2) is sparse due to the spatial- domain encoding, it can be further simplified as:

$$\mathbf{y} = \frac{1}{\sqrt{n_t}} \mathbf{h}_{k,\mathrm{eff}} + \mathbf{w}, \quad (3)$$

where $\mathbf{h}_{k,\mathrm{eff}}$ is referred to the effective channel vector consisting of the column vectors of $\mathbf{H}$ that are associated with the active antenna indices given by $\mathbf{h}_{k,\mathrm{eff}} = \sum_{j=1}^{n_t} \mathbf{h}_{k_j}$. Transmitted IC/SSK spatial symbols could be detected by the optimal maximum-likelihood (ML) detector as

$$\hat{k} = \arg \max_{\mathbf{s}_k \in \mathbb{S}} p\left(\mathbf{y}|\mathbf{H}\right) = \arg \min_{1 \le k \le N_t} ||\mathbf{y} - \frac{1}{\sqrt{n_t}} \mathbf{h}_{k,\mathrm{eff}}||_F^2 \quad (4)$$

where F denotes the Frobenius norm. Lastly, the transmitted information bits are recovered back by an inverse *one-to-one* mapping function between the coded and uncoded symbols.

A. Suboptimal Code Construction

To construct the optimal code set $\mathbb{C}_{\mathrm{opt}} = \{\mathbf{c}_1, \mathbf{c}_2, \cdots, \mathbf{c}_{N_t}\}$, the corresponding constellation symbols $\mathbf{u}_{k,\mathrm{eff}} = \mathbf{H}\mathbf{s}_k = \frac{1}{\sqrt{n_t}} \mathbf{h}_{k,\mathrm{eff}}$, $k = 1, 2, \cdots, M$, are chosen from $\mathcal{H} = \{\mathbf{h}_{1,\mathrm{eff}}, \cdots, \mathbf{h}_{M,\mathrm{eff}}\}$, that are widely spread-apart in the N_r-dimensional space, where $M = \binom{N_t}{n_t}$. Hence, an optimal and computationally feasible coding algorithm should maximize the minimum distance $d_{\min}$ according to

$$\mathbb{C}_{\mathrm{opt}} = \underset{g}{\operatorname{argmax}} \left\{ \min_{j,k,(j \ne k)} \left\{ d(j,k) = ||\mathbf{h}_{j,\mathrm{eff}}^{(g)} - \mathbf{h}_{k,\mathrm{eff}}^{(g)}||_F^2 \in \mathcal{H}_g \right\} \right\}.$$

where $\mathcal{H}_g = \{\mathbf{h}_1^{(g)}, \mathbf{h}_2^{(g)}, \cdots, \mathbf{h}_{N_t}^{(g)}\} \subseteq \mathcal{H}$ for $g = 1, 2, \cdots G$. Consequently, to determine the optimal code, we need to examine each possible code set containing N_t columns chosen from the available LED column combinations of $\mathcal{H}$. Since, the number of all possible code set combinations increases exponentially as a function of N_t and n_t, some suboptimal code selections algorithm needs to be designed with tolerable computational complexity. We now propose a new code selection algorithm that iteratively enlarges the minimum distance between N_r-dim real-valued vectors (points) in $\mathbb{S}$ until the number of selected points is reached that of the points in the codeword. Algorithm 1 describes the minimum distance code for construction for a given $\mathcal{H}$, N_t and n_t, in detail. Initially, through the nearest neighbor algorithm, the algorithm searches for the pairs of points that have the same minimum distance each. By computing the average distances of these pairs to the rest of the points, the pair, furthest distance from all the points, is then chosen for further processing. A point of the selected pair is then chosen as a candidate of a code word if its second nearest neighbor is further ahead of the other point's second nearest neighbor. This step continues in the following iterations until the algorithm chooses the last codeword. We expect that the codewords selected this way are maximally separated from each other and the minimum distance among the codewords is maximized.

B. Computational Complexity

The optimal code construction implemented by a standard brute forcing (BF) algorithm searches the maximum of the minimum distances in all possible subsets, $\mathcal{H}_m$ each consisting of N_t points, chosen from a given set of points, $\mathcal{H}$ in N_r-dim Euclidean space with cardinality $M = \binom{N_t}{n_t}$. The BF algorithm uses nested loops that requires $\frac{N_t(N_t-1)}{2} . \binom{M}{N_t} . N_r$

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Algorithm 1 **Min Dist Code Construction Algorithm**

Input: N_t, n_t, $\mathcal{H} = [\mathbf{h}_{1,\text{eff}}, \mathbf{h}_{2,\text{eff}}, \cdots, \mathbf{h}_{M,\text{eff}}]$
Output: $\mathbb{C}_{\text{opt}} = \{\mathbf{c}_0, \mathbf{c}_1, \cdots, \mathbf{c}_{N_t-1}\}$
Initialization $(j = 0)$
• Determine minimum distance pairs among the M N_r-dim points in $\mathcal{H}$ by the nearest neighbor algorithm(NN). $\mathcal{D} = \{d_{\text{min}}^{(n)}, n = 1, 2, \cdots, N_0\}$
• Choose a pair from $\mathcal{D}$, whose center point is maximally separated from all other points in $\mathcal{H}$, using the k-NN algorithm.
• Select a point of the chosen pair as an initial code word, $\mathbf{c}_0$, whose NN distance is larger than the other point.
while $j > N_t$ **do**
$j \leftarrow j + 1$
Repeat the same selection process as the initial step until all the N_t codewords $\mathbb{C} = \{\mathbf{c}_0, \mathbf{c}_1, \cdots, \mathbf{c}_{N_t-1}\}$ are selected.
end while

real multiplications (RMs) and real additions (RAs). It can be easily seen that even for small values of N_t and n_t, the complexity of the BF algorithm is enormous.

On the other hand, the suboptimal code construction algorithm proposed in this letter is based on finding the minimum distance pair in a given set of points and eliminating one of them iteratively until the total number of points is reduced to the number of the codewords, N_t. Hence, the main computational complexity to implement the algorithm comes from finding a minimum distance pair at each iteration step (j), in a set of points, $M^{(j)} = M - j, j = 0, 1, \cdots, N_t - 1$ with initially $M^{(0)} = M$. For this task, at each iterations step j, it requires $\frac{M^{(j)}(M^{(j)}-1)}{2}.N_r$ RMs and RAs=RMs-1. The multipler N_r represents the number of operations to compute Euclidean distance between two N_r-dim vectors. Consequently, the complete computational complexity of the algorithm in terms of RMs and RAs can be obtained as the algorithm chooses the last codeword. We expect that the codewords selected this way are maximally separated from each other and the minimum distance among the codewords is maximized.

$$\begin{aligned}\text{RMs} &= \left(\sum_{j=N_r+1}^{M} \frac{j(j-1)}{2}\right).N_r \\ &= \left(\frac{M(M-1)(M+1))}{6} - \frac{N_t(N_t-1)(N_t+1)}{6}\right).N_r \\ \text{RAs} &= \text{RMs} - 1.\end{aligned}$$

A comparison of BF with the proposed code construction algorithm is given in Table II, in terms of the number of RMs. As it can be seen from the table, the number of RMs and RAs increases exponentially very fast for the optimal algorithm, the increase is approximately $O(N.\log(N))$ for the proposed algorithm.

TABLE II. Comparison of Computational Complexity

Case	Suboptimal	Optimal
$Nt = 5, Nr = 2$	$145.Nr$	$2520.Nr$
$Nt = 6, Nr = 2$	$525.Nr$	$75075.Nr$
$Nt = 8, Nr = 3$	$29176.Nr$	$3.9774e + 10.Nr$
$Nt = 9, Nr = 5$	$333255.Nr$	$5.9279e + 14.Nr$

C. Bit Error Rate for Index-Coded SSK

Since exact analytical expressions for BER cannot be obtained in general, the performance is usually analyzed either by computer simulations and/or a tight upper bound on the bit error probability. Using well known union bounding technique based on pairwise error probabilities (PEP), the final BER upper bound for a IC/SSK system with parameters $\{N_t, N_r, n_t\}$, can be expressed as [1],

$$\text{BER} \leq \sum_j \sum_k \frac{N_{j,k}}{N_t} P(\mathbf{s}_j \rightarrow \mathbf{s}_k), \tag{5}$$

where $N_{j,k}$ is the number of bits in error between the coded constellation vectors $\mathbf{s}_j$ and $\mathbf{s}_k$, and $P(\mathbf{s}_j \rightarrow \mathbf{s}_k)$ denotes the PEP of deciding on $\mathbf{s}_k$ given that $\mathbf{s}_j$ is transmitted. Using (5), the PEP, condition on $\mathbf{H}$, is given by [1]

$$P(\mathbf{s}_j \rightarrow \mathbf{s}_k|\mathbf{H}) = Q(\sqrt{v}), \tag{6}$$

where $v = \frac{\rho}{n_t} d_{j,k}^2$, ρ being the signal-to-noise ratio defined as $\rho = E\{||\mathbf{h}_{j,\text{eff}}||_F^2\}/\sigma_w^2$. $d_{j,k}$ represents the Euclidean distance between $\mathbf{h}_{j,\text{eff}}$ and $\mathbf{h}_{k,\text{eff}}$ and $Q(x) = \int_x^\infty (1/\sqrt{2\pi}) e^{-t^2/2} dt$.

In order to show the system diversity and coding gain, we now derive a tight upper bound for the PEP based on the designed code set $\mathbb{C}_{\text{opt}}$, which is constructed to maximizes d_{min} over all $\binom{\binom{N_t}{n_t}}{N_t}$ possible code sets. Note that a looser upper bound is given in [1] for the BER. However this bound requires the exact knowledge of the number of distinct columns of $\mathbf{H}_{\text{eff}}$ between $\mathbf{h}_{\text{j, eff}}$ and $\mathbf{h}_{\text{k,eff}}$ for all j, k. The upper bound based on d_{min} of the code can be obtained as follows: From (6), we have $Q(\sqrt{v}) = Q\left(\sqrt{\frac{\rho}{n_t}} d_{j,k}\right)$. While the probabilty density function (pdf) of the random variable v is shown to be chi-squared distribution with $s = N_t$ degrees of freedom [7], exact analytical expression for the pdf of $\sqrt{v}$ is quite complicated. This is mainly due to the fact that although the components of the vector, $\boldsymbol{\alpha}_{j,k} = \mathbf{h}_{j,\text{eff}} - \mathbf{h}_{k,\text{eff}}$, are independent with zero-mean real Gaussian random variables, their variances depend on k and j through the number of distinct columns of $\mathbf{H}$. However, during the construction of the optimal code set, our extensive computer simulations showed that the pdf of d_{min} is fitted in a Gaussian distribution, perfectly with mean $\mu_{d_{\text{min}}}$ and variance $\sigma_{d_{\text{min}}}^2$. The mean and variance parameters are obtained as a by-product during the code construction. Since $Q(.)$ is monotonically decreasing and $d_{\text{min}} \leq d_{j,k}, \forall (j,k)$, a tight upper bound for the unconditional pair-wise error probability can be obtained as

$$P(\mathbf{c}_j \rightarrow \mathbf{c}_j) \leq \int_0^\infty Q(\sqrt{\frac{\rho}{2n_t}} x) p_{d_{min}}(x) dx \tag{7}$$

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where $p_{d_{\min}}(x) \sim \mathcal{N}\left(\mu_{d_{\min}}, \sigma^2_{d_{\min}}\right)$. The integral in (7) can be taken using standard calculus exactly and finally, an upper bound expression for the average BER follows as

$$\mathrm{BER} \leq C \times Q\left(\frac{\sqrt{\frac{\rho}{2n_t}}\mu_{d_{\min}}}{\sqrt{1+\frac{\rho}{2n_t}\sigma^2_{d_{\min}}}}\right), \tag{8}$$

where $C = \sum_{j=1}^{N_t-1}\sum_{k=j+1}^{N_t}\frac{N_{j,k}}{N_t}$.

III. Computer Simulations

In this section, BER versus signal-to-noise ratio (SNR) performance of optical IC/SSK and uncoded SSK systems are compared for varying parameters such as the number of LEDs (N_t), the number of randomly selected LEDs (n_t) and the number of PD's (N_r) at the receiver. The bound of (8) is also plotted for comparison. Based on [4], we adopt the configuration in which N_t, number of LEDs, are uniformly distributed over the ceiling of a 6 m × 6 m × 3 m room. It is assumed that each LED radiates 1 W optical power and the LED half-power semiangle is considered as $\Phi_{1/2} = 60^{\mathrm{o}}$. The channel gains of users are obtained by

$$h_{r,t} = \frac{(m+1)A_{\mathrm{PD}}}{2\pi\delta^2_{r,t}}\cos^m(\phi_{r,t})\cos(\theta_{r,t})\mathbb{1}_{\Psi_{1/2}}(\theta_{r,t}) \tag{9}$$

where $\phi_{r,t}$ and $\theta_{r,t}$ represent the angle of emergence and angle of incidence between the r^{th} receive PD and t^{th} LED, respectively. Also the distance between the r^{th} receive PD and t^{th} transmit light emitting diode (LED) is denoted by $\delta_{r,t}$. The parameter m is the Lambertian emission order of the light source and defined as $m = -1/\log_2(\cos(\Phi_{1/2}))$. A_{PD} denotes the effective area of the non-imaging photo-diode (PD). The indicator function $\mathbb{1}_{\Psi_{1/2}}(\cdot)$ determines whether the incidence angle is within the field-of-view (FoV) of the PD. The semiangle of the FoV and the physical area of each PD are chosen as $F_{\mathrm{PD}} = 70^{\mathrm{o}}$ and $A_{\mathrm{PD}} = 1\ \mathrm{cm}^2$, respectively. In this study, we consider an IC/SSK system operating in a 5m × 5m × 3 m typical room. We assume the LEDs on the ceiling with equidistant spacings of 2m, with a view angle of 120^o. PDs are located on a table with a height of 0.8m. The FoV semi-angle and area of PDs are 85^o and 1 cm^2, respectively. In computer simulations, we assume the average electrical signal-to-noise ratio at the receiver ($\mathrm{SNR_{Rx}}$) as given in [3], [8]:

$$\mathrm{SNR_{Rx}} = \frac{P^{\mathrm{elec}}_{Rx}}{\sigma^2_w} = \frac{\varrho(P^{\mathrm{opt}}_{Rx})^2}{\sigma^2_w} \tag{10}$$

where ϱ is the electrical-to-optical conversion factor that is taken as unity and P^{elec}_{Rx} is the average received electrical power. The averaged received optical power can be expresses as $P^{\mathrm{opt}}_{Rx} = (1/N_r)\sum_{r=1}^{N_r}\sum_{t=1}^{N_t} h_{r,t}I$, where I is the mean optical intensity. Note that the BER performances, obtained below, are based on the suboptimal IC/SSK codes. However, our computer simulations have shown that they are very close to that of the optimal ones.

Fig.1 illustrates the IC/SSK and uncoded SSK BER performances for $\eta = 2$ bits/sec/Hz spectral efficiency with $N_t = 4, n_t = 2$ and $N_r = 2, 3$. The upper bounds are also plotted in terms of the pre-determined means and variances. The IC/SSK's performance improvement, as well as the tightness of the upper bounds, are clearly seen in the figure. Namely, in Fig.1 we observe that without sacrificing the transmission rate, as opposed to the classical coded systems, effective coding gains of 5 dB and 9 dB are obtained over uncoded SSK at a target BER= 10^{-4} as the number of PDs increases from $N_r = 2$ to $N_r = 3$.

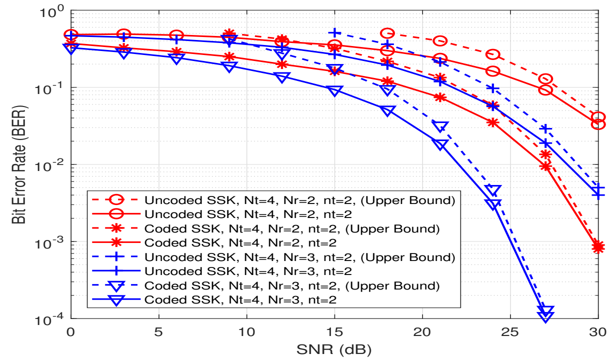

Fig. 1. BER performance of coded and uncoded SSK for $\eta = 2$ bits/sec/Hz transmission with $N_t = 4$, $N_r = 2, 3$ and $n_t = 2$ with the upper bounds

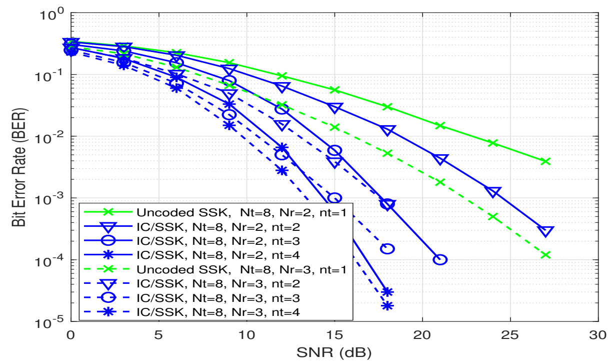

Fig. 2. BER performance of coded and uncoded SSK for $N_t = 8$ and for different N_r and n_t values.

In Fig. 2, BER performance of an IC/SSK system with 3 bits/sec/Hz spectral efficiency is investigated as the number of active LEDs (n_t) and PDs (N_r) varies for a given fixed $N_t = 8$. The IC/SSK's performance improvements are clearly seen in these plots as n_t increases from $n_t = 1$ to $n_t = \lfloor N_t/2 \rfloor = 4$.

In Fig. 3, an IC/SSK with $N_t = 8, n_t = 2, N_r = 2, 3$ is compared to a GSSK with $N_t = 5, n_t = 2, N_r = 2, 3$, both having the same 3 bits/s/Hz spectral efficiency. As seen in this figure, the BER performance of the IC/SSK is significantly better than both the uncoded and the GSSK schemes. Also, under equal spectral efficiency assumption, and for $N_r = 2, 3$, the BER performance of the uncoded SSK is better than that of the GSSK, mainly due to the created large-scale redundancy in index coding, as well as the inherent spatial diversity that exists in both systems.

A. BER Performance in the Presence of Blockage Channels

In VLC applications, there is always a possibility that optical channels between each LED at the transmitter and the

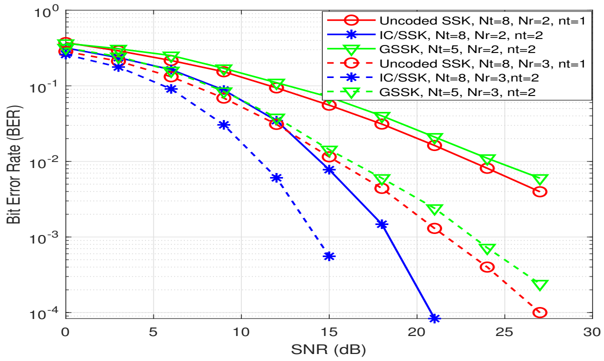

Fig. 3. BER performance comparisons of uncoded SSK, IC/SSK and GSSK for $\eta = 3$ bits/sec/Hz.

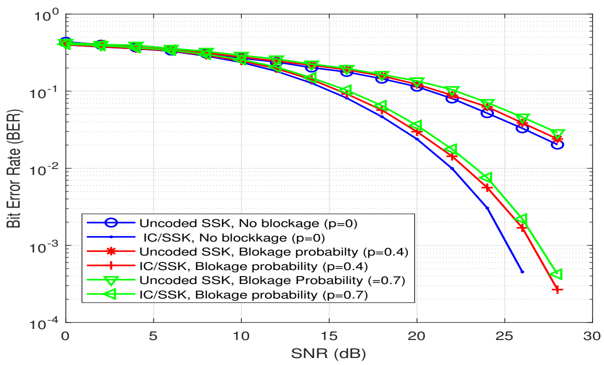

Fig. 4. BER performance of coded and uncoded SSK over blockage channels for $\eta = 3$ bits/sec/Hz, $N_t = 8, N_r = 4, n_t = 2$

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receiver can be partially blocked due to some blocking effects such as screening or some other reasons. Consequently, one or more PD outputs of the VLC system do not yield any useful information-bearing signal but simply a noise. The resulting VLC channel is called as *blockage channel*. We define the qth blockage status of the channel by the N_r-tuple binary vector $BL_q = [1,\ 1,\ 1,\ 0.\ \cdots,\ 1,\ 0]$ where 1 or 0 in the ith position of BL_q denotes,whether the ith PD is unblocked or blocked, respectively. Hence, depending on the blockage effect, we may have $2^{N_r} - 1$ different blockage configurations, excluding the "all blockage" scenario out of the model. Let p denotes the probability of any PD is blocked, independent of the other PD's. Then, Prob(no blocking) $= (1-p)^{N_r}$ and Prob(kPD's are blocked) $= \binom{N_r}{k} p^k (1-p)^{N_r-k}$. Similar to the CSI case, the blockage status of the channel can also be fed back to the transmitter to let the transmitter site choose the best index code set that fits in the blocked channels in the best way. Receiver can determine the blockage status of each channel between TX j and RX i by monitoring an indicator function based on the received signal-to-interference-plus-noise ratio at the phase detector (PD) i defined as [9],

$$SNR_i = \frac{\left(R_e \zeta \sum_{j=1}^{N_t} b_{j,i} H_{j,i} (I_{j,i}/2)^2\right)^2}{\sigma_n^2 + \left(R_e \zeta \sum_{k \neq i}^{N_r} \sum_{j=1}^{N_t} b_{j,i} H_{j,i} (I_{j,i}/2)^2\right)^2}$$

with R_e the responsitivity of the PD, ζ the energy conversion efficiency from electrical to optical power, σ_n^2 is the variance of the Gaussian noise and $I_{j,i}$ denotes the light intensity between ith LED and jth PD. Blockage state indicator $b_{j,i}$ is set to $b_{j,i} = 1$ if LOS path not blocked and $b_{j,i} = 0$ if LOS path is blocked. When one or more SNR_i indicators falls below a certain threshold, the CSI as well the blockage status can be sent back to the source by a feed back infrared (IR) channel, to adjust the index coding at the transmitter site.

To demonstrate how this new technique works, we now present a VLC system with $N_t = 8$, $N_r = 4$ and $n_t = 2$, operating over a VLC blockage channel. The system BER performance is studied for different probabilities of any PD is blocked as $p = 0, 0.4, 0.7$. Fig. 4 illustrates the BER vs SNR performance of the uncoded and IC/SSK-based VLC system with these parameters. As seen from this figure, IC/SSK is very effective to combat blockage channels and can achieve around 12 dB coding gains at a target BER=10^{-3}, again without reducing the bit rate of the system.

IV. Conclusions

A new channel coding technique was proposed for optical SSK systems by introducing redundancy in the spatial domain. Depending on the number of LEDs and the PDs as well as the amount of redundancy, it was concluded that the IC/SSK achieved excellent coding gains in full rate at given target BER values, as compared to the conventional channel coding in bit-domain, and yielded superior BER performance in the presence of the VLC blockage channels, as compared to the uncoded SSK systems.

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