

# Generation/transmission investment planning integrated with market equilibrium models in electricity markets

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**Abstract**— This paper introduces integrated models for transmission investments anticipating the generation investments and market-clearing equilibrium. Market-clearing models for deregulated electricity markets can inform decision makers on price signals formed in the competitive market, other investor's and/or generator's behaviors underlying these price signals and new generation/transmission investment decisions. Bi-level programming problems are formed for this integrated models and reformulated by using mathematical programs with equilibrium constraints (MPEC). A simultaneous optimization model (as mixed complementarity problem –MCP) is proposed to compute the same equilibrium solution of the MPEC problem. The proposed MCP model is found to be computationally more efficient than the traditional MPEC reformulations on a simple 3-bus example. These models will be useful in planning generation/transmission investments, and analyzing the relations among these investments and the market outcomes.

**Index Terms**—Generation and transmission investment models, mathematical programs with equilibrium constraints (MPEC), simultaneous optimization by mixed complementarity problem (MCP).

## I. INTRODUCTION

In the organized electricity markets, planning and investment decisions of the privately owned generation companies are driven by economic considerations in response to market outcomes. On the other hand, investment decisions for the transmission system are anticipated by the transmission system operator (TSO)<sup>1</sup> who characterizes reliable and secure market operations [1]. Clearly, planning and investment in generation and transmission as well as market-clearing procedures are closely related and influenced by a variety of factors including fuel costs, behavior of the generation companies, network topology, demand response and uncertainties in demand and generation assets. Hence, a requirement for the deregulated markets is the integration of

the models for generation/transmission investments and market-clearing.

Market-clearing models for deregulated electricity markets can inform decision makers on price signals formed in the competitive market, other investor's and/or generator's behaviors underlying these price signals and new generation/transmission investment decisions. Moreover, uncertainty in supply-demand dynamics that accounts for generation costs and demand response can be analyzed. These models can also include realistic features such as linear models of flows on high-voltage transmission lines (lossless and approximate dc flows) and stochastic features to account for generation/demand uncertainties.

In this context, such models would have an important role in the decision making process by revealing the complex market dynamics. The proposed model of this paper will be useful in terms of future investment plans and their impact on the market and decision-making process of market participants, as well as this will enable analyses of the market participants' short/medium/long-term decisions. Moreover, large-scale but simplified models (e.g., models that represent roughly the capacity of a region but only a simplified transmission network) may be used to suggest basic pricing/investment results. This type of research can alert decision makers to the need for market policies generally and to examine the interaction between some elements of market design and market outcomes (see [2] for an account).

Such models are used, and could be used further, in regulatory decision-making and antitrust oversight. For example, fairly detailed regional models are used in the screening of electricity industry merger applications, to define geographical antitrust markets and calculate concentration indices. Finally, there are market power simulations that attempt to short-term forecast or ex-post replicate the actual hourly prices that result in a market. These models have emerged most recently in the effort to understand the role of market power in the California market [3, 4] as well as to replicate spot market prices in New England [5].

In this paper, formulation of a market-clearing/market-price simulation model for electricity markets and integration

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<sup>1</sup>In general, the market operator and the system operator may be different independent entities, but we assume that they can have close coordination for market efficiency as well as secure and reliable system operations.

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of the generation/transmission investment models into this model is proposed. For this integrated model, a simultaneous optimization model is formulated as a mixed complementarity problem (MCP) and it is compared to the bi-level model results formulated as mathematical programs with equilibrium constraints (MPEC). The benefit of MCP formulation is faster and reliable computation of solutions compared to MPEC reformulation. Proposed models are examined on a simple 3-bus test system.

The contributions of this paper are twofold: 1) We present and analyze two different formulations (a single level problem formulated as MCP and a bi-level problem formulated as MPEC) for integrated transmission investment planning with generation investments and market-clearing equilibrium; 2) the challenges in solving the bi-level problem using MPEC reformulations are addressed.

The remainder of the paper is organized as follows. In section II, background on integrated transmission/generation investment models and market-clearing equilibrium is presented. Section III describes the mathematical models and presents the formulation of MCP and MPEC models. Section VI presents an illustrative example and numerical results. The paper is finalized with section V, where conclusions and directions for future research are summarized.

## II. BACKGROUND

In the literature, there are many studies that formulates an “optimal” transmission and/or generation investment planning where single decision maker’s investment decisions are in the upper level(s) with system operator’s market-clearing problem as a lower level subproblem. These bi-level (multi-level) problems are particularly relevant for policy analysis when the leader agent(s) influences market equilibrium by anticipating the decisions of followers at the lower level. A summary of traditional and market based transmission investment methodologies were presented [6]. Market-based transmission and generation investment models are included in [7-11]. Bi-level programming models were presented in [12-16].

Sauma and Oren [17, 18] presented a generation investment planning model for multiple generation firms that anticipate the system operator’s market-clearing problem. They have investigated the social welfare effects of different transmission investment alternatives. However, in their model the transmission investment plan is a parameter rather than a decision variable. A three-level static generation and transmission investment model with market-clearing problem is presented in [19] with a reformulation that converted the three-level problem into a single level mixed integer linear programming problem (MILP). However, they do not consider the demand as a decision variable in their study (i.e., no market-clearing equilibrium).

Other studies use iterative methods [20], evolutionary algorithms [21, 22], agent-based system and search-based techniques [23] as solution approaches to similar bi-level or multi-level problems.

In [24], the authors formulate a three-level problem where two lower level problems (market-clearing and generation investments) form an equilibrium problem with equilibrium

constraints (EPEC). This EPEC sub-problem is solved by either the diagonalization method (DM) or a complementarity problem (CP) reformulation. Then, they propose a hybrid iterative solution algorithm that combines the CP reformulation of the three-level problem and DM solutions. However, they do not present any convergence analysis for their algorithms.

In [25], open and closed loop models for bi-level and simultaneous optimization problems are presented and analyzed. Open loop model represents a single level situation where investment decisions and market equilibrium are decided simultaneously. On the other hand, the closed loop model describes a bi-level problem where investment decisions are made in the upper level and market equilibrium is in the lower level. Considering a single period, they have found that the closed loop equilibrium for any strategic market behavior yields the open loop Cournot outcomes.

In this context, our results support the findings in [25] and presents that several investment decisions made in the upper level model may not be computationally challenging to solve. Many integrated models in the literature can be solved by a single level problem (instead of a bi-level or three-level one) by expanding the framework of the market-clearing equilibrium problem in a way that can include generation and transmission investment models. Considering the computational burden on bi-level and three-level formulations, this approach is more viable and also it can be extended to discrete investment decisions, i.e., complementarity problems with binary variables (see [26] for details). For example, stochastic bi-level programming models proposed by [27] where TSO first decides on transmission investment followed by the generation firms’ investment, decisions and market equilibrium, presents these challenges for large-scale models.

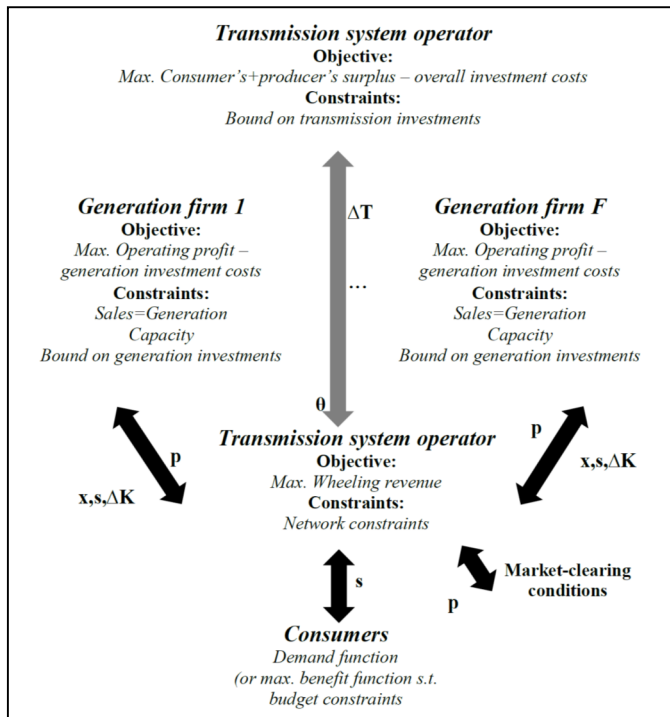


Figure 1: Integrated model of transmission/generation investment planning with market-clearing equilibrium

Figure 1 presents the single level integrated model proposed, TSO's and generation firms' investment decisions and market-clearing equilibrium.

An up-to-date assessment of the present and potential capabilities of such integrated models is presented in [28]. They have coined the term "co-optimization" for similar integrated problems in electricity markets and discussed needs and challenges for developing similar advanced models.

### III. GENERATION/TRANSMISSION INVESTMENT PLANNING INTEGRATED WITH MARKET EQUILIBRIUM MODELS

In this section we have explained the details of the mathematical model for the integrated generation/transmission investment and market equilibrium. For simplicity, we have formulated a static and deterministic model with a single period (e.g., hourly operation) for a target planning year, as similar to the approaches in the literature. We have assumed all investment costs are discounted to form equivalent hourly costs. But in our numerical examples, we have illustrated the model for a year by assuming all 8760 hours are identical (e.g., for sake of simplicity all operation costs are summed over 8760 hours of a year and investment cost parameters are yearly discounted rates). Below, we introduce the notation for the model in details.

#### Indices:

$f \in F$ , set of generation firms

$i, j \in I$ , set of buses (nodes)

$J_i \subset I$ , set of buses connected to bus  $i$

$I_f \subset I$ , set of generators owned by firm  $f$  at bus  $i$

#### Variables:

$p_i$  electricity (nodal) price at bus  $i$

$x_{fi}$  generation by firm  $f$  at bus  $i$

$s_{fi}$  sales by firm  $f$  at bus  $i$

$\theta_i$  voltage angle of bus  $i$

$\Delta T_{ij}$  new transmission investment on generation  $i$

$\Delta K_{fi}$  new generation investment by firm  $f$  at bus  $i$

$T_k$  upper level to the flow through transmission line  $k$ , after transmission system operator's (TSO) decisions made at the upper level

#### Parameters:

$\alpha_i$  non-price effects at bus  $i$  for the linear inverse demand function (weather, socio-demographic factors, etc.)

$\beta_i$  constant price coefficient for the linear inverse demand function at bus  $i$

$c_{fi}$  operating cost of generation firm  $f$  at bus  $i$

$c_{fi}^{Gexp}$  investment cost of new generation capacity for firm  $f$  at bus  $i$

$c_{ij}^{Texp}$  investment cost of new transmission capacity for transmission line connecting buses  $i$  and  $j$

$K_{fi}^0$  capacity of generation firm  $f$  at bus  $i$  before generation firm's investment decisions

$K_{fi}^{max-exp}$  generation firm  $f$ 's maximum investment level at bus  $i$

$B_{ij}$  susceptance of transmission line connecting buses  $i$  and  $j$

$T_{ij}^0$  upper level to the flow through transmission line connecting buses  $i$  and  $j$  before TSO's investment decisions.

$T_{ij}^{max-exp}$  maximum investment level for transmission line connecting buses  $i$  and  $j$ .

#### A. TSO's and Generation Firms' Investment Models and Market-Clearing Equilibrium

This market model is due to [29] and more generally due to [30] (but using bus angles as decision variables instead of power transfer distribution factors –PTDFs and net nodal injections/withdrawals in TSO's problem). According to this model, we introduce an elementary model of perfect and imperfect (oligopolistic) competition among generation firms. The following model is an open-loop perfectly competitive model where each agent in the model jointly solves their decision problem.

The response of the consumers to price changes is formulated by an inverse demand function (i.e., a utility maximization problem for the consumers is represented by this demand equation). Each bus in the transmission network has its own linear inverse demand, depending on each generation firm's sales at this bus in the equilibrium solution.

$$f_{d,i}^{-1} \left( \sum_{f \in F} s_{fi} \right) = \alpha_i - \beta_i \left( \sum_{f \in F} s_{fi} \right) \quad \forall i \in I \quad (1.1)$$

Generating firm  $f$  is a price-taker (for the perfect competition model) and it views price at each and every bus as an exogenous parameter in its objective function, even though from the market's point of view those prices are variable and are adjusted to balance supply and demand at each bus. Each generator firm maximizes its profit (revenues minus operating minus investment costs) subject to total sales equal total generation constraints and upper bounds on generation amounts/generation expansion, where  $\Delta K_{fi}$  represent the generation investments by firm  $f$  (which also affects the upper bound on generation amount):

$$\begin{aligned} \min_{s_{fi}, x_{fi}, \Delta K_{fi}} & - \sum_{i \in I_f} \left( \alpha_i - \beta_i \left( \sum_{f \in F} s_{fi} \right) \right) s_{fi} + \sum_{i \in I_f} c_{fi} x_{fi} \\ & + \sum_{i \in I_f} c_{fi}^{Gexp} \Delta K_{fi} \end{aligned} \quad (1.2)$$

$$\begin{aligned} \text{s.t.} \\ \sum_{i \in I} s_{fi} - \sum_{i \in I_f} x_{fi} = 0 \quad (v_f) \quad \forall f \in F \end{aligned} \quad (1.3)$$

$$x_{fi} \leq K_{fi}^0 + \Delta K_{fi} \quad (\mu_{fi}) \quad \forall f \in F, i \in I_f \quad (1.4)$$

$$\Delta K_{fi} \leq K_{fi}^{max-exp} \quad (\delta_{fi}) \quad \forall f \in F, i \in I_f \quad (1.5)$$

$$s_{fi} \geq 0, \quad x_{fi} \geq 0, \quad \Delta K_{fi} \geq 0 \quad \forall f \in F, i \in I_f \quad (1.6)$$

TSO's objective is to allocate transmission capacity to maximize the value that the market receives from network assets subject to network constraints. This can be shown to be equivalent to having the TSO choose values to maximize its

revenue and a competitive market for transmission rights in which generators do not exercise market power [30] (i.e., TSO acting as a market arbitrageur, see [29] for details). Moreover, TSO also decides on transmission expansion levels where  $\Delta T_{ij}$  represents the transmission investment decisions (which also affect the upper bound on power flows) subject to upper bounds on transmission expansion:

$$\min_{\theta_i, \Delta T_{ij}} - \sum_{i \in I} \left( p_i \sum_{j \in J_i} B_{ij}(\theta_i - \theta_j) \right) + \sum_{j \in J_i} c_{ij}^{Texp} \Delta T_{ij} \quad (1.7)$$

s.t.

$$B_{ij}(\theta_i - \theta_j) \leq T_{ij}^0 + \Delta T_{ij} \quad (\lambda_{ij}^+), \quad \forall i \in I, j \in J_i \quad (1.8)$$

$$-B_{ij}(\theta_i - \theta_j) \leq T_{ij}^0 + \Delta T_{ij} \quad (\lambda_{ij}^-), \quad \forall i \in I, j \in J_i \quad (1.9)$$

$$\Delta T_{ij} \leq T_{ij}^{max-exp} \quad (\gamma_{ij}), \quad \forall i \in I, j \in J_i \quad (1.10)$$

$$-\pi \leq \theta_i \leq \pi \quad (\varepsilon_i^{min}, \varepsilon_i^{max}), \quad \forall i \in I \quad (1.11)$$

$$\theta_i = 0 \quad (\xi), \quad i = \text{slack bus} \quad (1.12)$$

$$\Delta T_{ij} \geq 0 \quad \forall i \in I, j \in J_i \quad (1.13)$$

TSO's behavior is represented by modeling her as a market agent who believes it can not affect the market price, even though in fact price is a variable that is endogenous to the market. Basically TSO operates a market that auctions off the capacity of individual transmission components. It's similar to the "flowgate" market which is proposed by Chao and Peck (see [29, 30]).

$$\sum_{f \in F} x_{fi} - \sum_{f \in F} s_{fi} - \sum_{j \in J_i} B_{ij}(\theta_i - \theta_j) = 0 \quad (p_i) \quad \forall i \in I \quad (1.14)$$

Basically, the market-clearing conditions (1.14) depend on supply and demand balance at each bus  $i$  and, in the complementarity problem, are associated with the nodal price ( $p_i$ ).

The market equilibrium problem consists of the first order conditions (i.e., Karush-Kuhn-Tucker -KKT) for the generation firms' and TSO's problem together with the market clearing condition (1.14), which can be cast as the MCP in (1.15). Note that the non-negativity constraints (and their duals) are omitted from MCP (1.15) and therefore the corresponding KKT conditions are in " $\geq$ " form.

It is also possible to model imperfect competition (e.g., Cournot) among generator firms in this framework by adding the term,  $\beta_i s_{fi}$ , to the first condition in (1.15), i.e., all firms are aware of the price-quantity relations (i.e., demand function) in the market. Also note that, the transmission and generation investments are represented within (1.15). Instead of anticipating the market outcomes, the TSO and the generation firms simultaneously solve for their investment problems within the market-clearing equilibrium problem. Similar to approaches in [19], [24] and [27], it can be modeled separately

(as a bi-level or three-level over the market equilibrium model).

MCP: Find  $s_{fi}, x_{fi}, \Delta K_{fi}, v_f, \mu_{fi}, \delta_{fi}, \theta_i, \Delta T_{ij}, \lambda_k^+, \lambda_k^-, \gamma_{ij}, p_i, \varepsilon_i^{min}, \varepsilon_i^{max}, \xi$  such that

$$\begin{aligned} s_{fi} \geq 0 \perp & \quad -\alpha_i + \beta_i \left( \sum_{f \in F} s_{fi} \right) & \quad \forall f \in F, \\ & \quad + p_i + v_f \geq 0 & \quad i \in I \\ x_{fi} \geq 0 \perp & \quad c_{fi} - p_i - v_f + \mu_{fi} \geq 0 & \quad \forall f \in F, \\ & & \quad i \in I_f \\ \Delta K_{fi} \geq 0 \perp & \quad c_{fi}^{Gexp} - \mu_{fi} + \delta_{fi} \geq 0 & \quad \forall f \in F, \\ & & \quad i \in I_f \\ v_f \text{ free} \perp & \quad \sum_{i \in I} s_{fi} - \sum_{i \in I_f} x_{fi} = 0 & \quad \forall f \in F \\ \mu_{fi} \geq 0 \perp & \quad x_{fi} \leq (K_{fi}^0 + \Delta K_{fi}) & \quad \forall f \in F, \\ & & \quad i \in I_f \\ \delta_{fi} \geq 0 \perp & \quad \Delta K_{fi} \leq K_{fi}^{max-exp} & \quad \forall f \in F, \\ & & \quad i \in I_f \\ \theta_i \text{ free} \perp & \quad \sum_{j \in J_i} B_{ij}(p_i - p_j) \\ & \quad + \sum_{j \in J_i} B_{ij}(\lambda_{ij}^+ - \lambda_{ij}^-) & \quad \forall i \in I \\ & \quad - \sum_{j \in J_i} B_{ij}(\lambda_{ij}^- - \lambda_{ij}^+) & \\ & \quad + \varepsilon_i^{max} - \varepsilon_i^{min} + \xi = 0 & \quad (1.15) \\ \Delta T_{ij} \geq 0 \perp & \quad c_{fi}^{Texp} - \lambda_{ij}^+ - \lambda_{ij}^- - \gamma_{ij} \geq 0 & \quad \forall i \in I, \\ & & \quad j \in J_i \\ \lambda_{ij}^+ \geq 0 \perp & \quad B_{ij}(\theta_i - \theta_j) \leq T_{ij}^0 + \Delta T_{ij} & \quad \forall i \in I, \\ & & \quad j \in J_i \\ \lambda_{ij}^- \geq 0 \perp & \quad -B_{ij}(\theta_i - \theta_j) \leq T_{ij}^0 + \Delta T_{ij} & \quad \forall i \in I, \\ & & \quad j \in J_i \\ \gamma_{ij} \geq 0 \perp & \quad \Delta T_{ij} \leq T_{ij}^{max-exp} & \quad \forall i \in I, \\ & & \quad j \in J_i \\ p_i \text{ free} \perp & \quad \sum_{f \in F} x_{fi} - \sum_{f \in F} s_{fi} \\ & \quad - \sum_{j \in J_i} B_{ij}(\theta_i - \theta_j) = 0 & \quad \forall i \in I \\ \varepsilon_i^{min} \geq 0 \perp & \quad -\theta_i \leq \pi & \quad \forall i \in I \\ \varepsilon_i^{max} \geq 0 \perp & \quad \theta_i \leq \pi & \quad \forall i \in I \\ \xi \text{ free} \perp & \quad \theta_{i=\text{slack bus}} = 0 \end{aligned}$$

### B. Transmission Investment Model Anticipating the Generation Firms' Investment Models and Market-Clearing Equilibrium

Alternatively, a bi-level formulation can be formed where TSO's investment decisions are at the upper level and the generation expansion decisions at the lower level along with

the market equilibrium. In this case, TSO decides on transmission expansion plans for a set of candidate lines and anticipates the market equilibrium and generation investment model results. TSO maximizes the overall net surplus (consumer's and producer's surpluses, e.g., total surplus, minus investment costs subject to upper bounds on transmission investment level, which forms the following MPEC.

$$\begin{aligned} \min_{\Delta T_{ij}} & - \sum_{i \in I} \left( \alpha_i \left( \sum_{f \in F} S_{fi} \right) - \frac{1}{2} \beta_i \left( \sum_{f \in F} S_{fi} \right)^2 \right) \\ & + \sum_{i \in I_f} c_{fi} x_{fi} + \sum_{i \in I_f} c_{fi}^{Gexp} \Delta K_{fi} \\ & + \sum_{i \in I, j \in J_i} c_{ij}^{Texp} \Delta T_{ij} \end{aligned} \quad (1.16)$$

s.t.

$$\Delta T_{ij} \leq T_{ij}^{max-exp} (\gamma_{ij}), \quad \forall i \in I, j \in J_i \quad (1.17)$$

$$\Delta T_{ij} \geq 0 \quad \forall i \in I, j \in J_i \quad (1.18)$$

$$\text{MCP (1.15) excluding conditions for } \Delta T_{ij} \text{ and } \gamma_{ij} \quad (1.19)$$

MCP model (1.19) can also be converted to a set of constraints (including several binary variables) by using Fortuny-Amat & McCarl [31] method, which is a widely used approach for solution of bi-level as well as three-level problems. However, the main drawback is that the number of binary variables introduced for this reformulation is usually large and this may cause computational issues. Also choice of big  $M$  is not trivial. Suppose  $a$  and  $b$  as variable-condition pairs:

$$a \geq 0 \perp b \geq 0 \quad (\text{or } a \geq 0, b \geq 0, a \cdot b = 0)$$

Each variable-condition pair can be replaced by the following constraints and binary variables  $u$  (where  $M$  is large positive scalar):

$$a \leq Mu, b \leq M(1 - u), a \geq 0, b \geq 0, u \in \{0,1\}.$$

Using this reformulation, MPEC problem becomes a mixed integer quadratic program (MIQP) and it can be solved by the state of the art CPLEX solver. The uniqueness and existence of solutions of the MCP problem (1.19) can be verified. Continuous and decreasing (i.e., monotone) demand functions ( $\beta_i > 0$ ) and strictly convex cost functions for generating firms are sufficient to ensure that a solution exists and that the quantities and prices are unique. But the MCP problem included in transmission investment model by Fortuny-Amat & McCarl method, does not guarantee existence and uniqueness (i.e., overall MPEC is non-convex). Hence, the reformulation does not guarantee a global (or local solution), but just stationary solutions to the problem, if one exists.

#### IV. ILLUSTRATIVE EXAMPLE

The proposed models are tested on a simple 3-bus example from [30], where firm 1 is located at bus 1 and firm 2 is located at bus 2. Most of the data is adapted from [30] and all parameters are detailed in Table 1 (generation and

transmission expansion cost parameters are discounted assuming a lifetime of 20 years and an interest rate of 3% per year).

The models are solved using GAMS (PATH solver for MCP model, NLPEC for the MPEC model and CPLEX for the MIQP model) on a personal computer with a 2.3 GHz processor and 4GB RAM. As this is a small-scale illustrative example, the solution times are less than a second for the MCP model (0.047 seconds) and for the MPEC model (0.297 seconds). Also the reformulated MPEC model as MIQP has slightly lower computation time than the MPEC model (0.203 seconds), but the feasibility of the problem is very much dependent on the choice of big  $M$ , i.e., too large or small values may cause higher computation times or infeasibilities/unboundedness, and usually a trial and error approach is required (as bounds on some dual variables are not always easy to estimate). All the results are identical for MCP, MPEC and MIQP models and are summarized in Table 2. This indicates that the MCP model seems computationally more efficient than the MPEC and MIQP reformulations. A large-scale problem implementation (e.g., a stochastic model or a model with a larger network) would reveal the advantages more clearly.

Table 1: Model parameters

<b>Generation parameters</b>	<b>Firm 1</b>	<b>Firm 2</b>	
$c_{fi}$ (\$/MWh)	15	20	
$c_{fi}^{Gexp}$ (\$/MW/year)	15,000	12,000	
$K_{fi}^0$ (\$/MW)	480	350	
$K_{fi}^{max-exp}$ (MW)	100	100	
<b>Demand parameters</b>	<b>Bus 1</b>	<b>Bus 2</b>	<b>Bus 3</b>
$\alpha_i$ (\$)	40	40	40
$\beta_i$ (\$/MW)	0.08	0.08	0.0516
<b>Line parameters</b>	<b>Line 1-2</b>	<b>Line 1-3</b>	<b>Line 2-3</b>
$c_{ij}^{Texp}$ (\$/MW/year)	27,000	N/A	N/A
$T_{ij}^0$ (MW)	25	1000	1000
$T_{ij}^{max-exp}$ (MW)	50	N/A	N/A
reactance (p.u.)	0.1	0.1	0.1

Table 2: Summary of MPEC & MCP Model Results

$p_1$	16.71	$Sales_1$	291.10	$x_{11}$	535.80
$p_2$	20.00	$Sales_2$	250.00	$x_{22}$	269.71
$p_3$	18.36	$Sales_3$	264.42		
$\Delta T_{1-2}$	50	$Flow_{1-2}$	75	$Profit_1$	8,037K*
$\Delta K_{11}$	55.80	$Flow_{1-3}$	169.71	$Profit_2$	0
$\Delta K_{22}$	0	$Flow_{2-3}$	94.71	$NetProfit_1$	7,200K
<b>Trans. Rev</b>	<b>3,240K</b>		<b>Cons. Surplus</b>		<b>67,393K</b>
<b>Trans. Exp. Cost</b>	<b>1,350K</b>		<b>Total Surplus</b>		<b>78,670K</b>
<b>Gen. Exp. Cost</b>	<b>837.0K</b>		<b>Net Surplus</b>		<b>76,483K</b>

\*where  $K=1,000$

#### V. CONCLUSIONS AND FUTURE RESEARCH

The proposed models in this paper consider an integrated transmission and generation investment model with market-clearing equilibrium. A simultaneous optimization model (MCP) is formed and compared to the results of the bi-level

reformulations (MPEC and MIQP), where a centralized TSO's decisions in the upper level is made in anticipation of the lower level generation firms' investment decisions and market-clearing equilibrium. The computational challenges for MPEC model and its reformulation as MIQP are also addressed. The advantages of the MCP model are presented on a simple 3-bus example.

For future research, we would like to extend this model framework by uncertainties in some model parameters (e.g., demand parameters, generator capacities) and different market structures (e.g., Nash-Cournot) to exploit the computational advantages of MCP formulation over a stochastic MPEC counterpart. Furthermore, comparison to three-level problems and analyses of computational results would be useful.

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