Joint Channel Estimation and Equalization for OFDM based Broadband Communications in Rapidly Varying Mobile Channels

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***Abstract*—This paper is concerned with the challenging and timely problem of channel estimation for orthogonal frequency division multiplexing (OFDM) systems in the presence of frequency selective and very rapidly time varying channels. In OFDM systems operating over rapidly time-varying channels, the orthogonality between subcarriers is destroyed leading to inter-carrier interference (ICI) and resulting in an irreducible error floor. The band-limited, discrete cosine serial expansion of low-dimensionality is employed to represent the time-varying channel. In this way, the resulting reduced dimensional channel coefficients are estimated iteratively with tractable complexity and independently of the channel statistics. The algorithm is based on the expectation maximization-maximum a posteriori probability (EM-MAP) technique leading to a receiver structure that also yields the equalized output using the channel estimates. The pilot symbols are employed to estimate the initial coefficients effectively and unknown data symbols are averaged out in the algorithm in a non-data-aided fashion. It is shown that the computational complexity of the proposed algorithm to estimate the channel coefficients and to generate the equalized output as a by-product is $\sim O(N)$ per detected symbol, N being the number of OFDM subcarriers. Computational complexity as well as computer simulations carried out for the systems described in WiMAX and LTE standards indicate that it has significant performance and complexity advantages over existing suboptimal channel estimation and equalization algorithms proposed earlier in the literature.**

I. Introduction

Orthogonal frequency-division multiplexing (OFDM) with a cyclic prefix (CP) has been shown to be an effective method to overcome inter-symbol interference (ISI) effects due to frequency-selective fading with a simple transceiver structure. Consequently, it is becoming a key air interface of next-generation wireless communications systems such as the IEEE 802.16 family - better known as Mobile Worldwide Interoperability Microwave Systems for Next-Generation Wireless Communication Systems (WiMAX) - and by the Third-Generation Partnership Project (3GPP) in the form of its Long-Term Evolution (LTE) project. OFDM eliminates ISI and simply uses a one-tap equalizer to compensate for multiplicative channel distortion in quasi-static channels. However, in fading channels with very high mobility, the time variation of the channel over an OFDM symbol period results in a loss of subchannel orthogonality which leads to inter-carrier interference (ICI). Since mobility support is widely considered to be one of the key features in wireless communication systems, and in this case ICI degrades the performance of OFDM systems, OFDM transmission over very rapidly time varying multipath fading channels has been considered in a number of recent works [1], [2], [3], [4].

To reduce the effects of ICI, a time-domain channel estimator was proposed in [1] which assumed that the channel impulse response (CIR) varies in a linear fashion within the symbol duration. However, this assumption no longer holds when the normalized Doppler frequency takes substantially higher values. In a rapidly time-varying channel, the time-domain channel estimation method proposed in [5] is a potential candidate for the channel estimator, in order to mitigate ICI. This technique estimates the fading channel by exploiting the time-varying nature of the channel as a provider of time diversity and reduces the computational complexity using the singular-value decomposition (SVD) method. In [3], to handle rapid variation within an OFDM symbol, the pilot-based estimation scheme using channel interpolation was proposed. Moreover, coupled with the proposed channel estimation scheme, a simple Doppler frequency estimation scheme was proposed.

In [4], two methods to mitigate ICI in an OFDM system with coherent channel estimation were proposed. Both methods employed a piece-wise linear approximation to estimate channel time-variations in each OFDM symbol. The first method extracted channel time-variation information from the cyclic prefix while the second method estimated these variations using the next symbol. Moreover, a closed-form expression for the improvement in average signal-to-interference ratio (SIR) was derived for a narrowband time-varying chan-

This research has been conducted within the NEWCOM++ Network of Excellence in Wireless Communications and WIMAGIC Strep projects funded through the EC 7th Framework Programs and was supported in part by the U.S. National Science Foundation under Grant CNS-09-05398.

nel.

Recently in [6] and [7], a joint channel estimation, equalization and data detection algorithm has been presented for OFDM systems in the presence of high mobility channels based on the space alternating generalized expectation maximization (SAGE) technique. However, the main objective of this work was equalization and detection of data symbols rather that estimating the channel coefficients directly. The channel estimates are obtained as a byproduct of the algorithm. Therefore, it is computationally more intensive.

In this work a new expectation-maximization/maximum a posteriori probability (EM-MAP) based iterative channel estimation algorithm is proposed for OFDM systems operating over rapidly varying frequency selective channels in a non-data-aided fashion. The main novelty of the paper comes from the facts that [1] the estimation is performed in the time-domain so that unknown data can be averaged out easily in the resulting algorithm since the OFDM data samples transmitted in the time-domain are approximately Gaussian distributed random variables, and [2] the proposed algorithm leads to a receiver structure that yields also an equalized output from which the data symbols are detected with an excellent symbol error rate (SER) performance. In order to reduce the large number of unknown channel coefficients, the band-limited, discrete orthogonal cosine transform (DCT) basis functions are employed to represent a time-varying fading channel having Jake's Doppler profile. The DCT basis functions are well suited to describe such a low-pass channel and have also the advantage of being independent of the channel statistics. Consequently, it is seen that only a few channel coefficients need to be estimated iteratively with tractable complexity. Exploiting block diagonal as well as banded structures of several matrices involved in the proposed algorithm, the computational complexity of the algorithm is shown to be $\sim O(N)$ per detected data symbol where N is the number of OFDM subcarriers. Computer simulations performed for OFDM based systems described in WiMAX and LTE standards show that the algorithm has significant performance and complexity advantages over existing algorithms.

II. Signal Model

We consider an OFDM system with N subcarriers. At the transmitter, K out of N subcarriers are actively employed to transmit data symbols and nothing is transmitted from the remaining $N-K$ carriers. The time-domain transmitted signal is denoted as

$$s(n) = \frac{1}{N}\sum_{k=0}^{K-1} d(k)\, e^{j2\pi nk/N} \;,\; n = 0, 1, \cdots, N-1 \;, \quad (1)$$

where n and k are the discrete-time and the discrete-frequency indices, respectively. $d(k)$ stands for the frequency domain data symbol transmitted over the kth OFDM subchannel. By the central limit theorem, the transmitted signal $s(n)$ can be modelled as a zero-mean colored complex Gaussian sequence provided K is sufficiently large. A cyclic prefix of length L_c is then added. We assume a time-varying multipath mobile radio channel with discrete-time impulse response $h(n,\ell)$, $\ell = 0, 1, \cdots, L-1$ where L is the maximum channel length and it is assumed that $L \leq L_c$. At the receiver, after matched filtering, symbol-rate sampling and discarding the symbols falling in the cyclic prefix, the received signal at the input of the discrete Fourier transform (DFT) can be expressed as

$$r(n) = \sum_{\ell=0}^{L-1} h(n,\ell)\, s(n-\ell) + w(n) \;,\; n = 0, 1, 2, \ldots, N–1 \;, \quad (2)$$

where $w(\cdot)$ is zero-mean complex additive Gaussian noise with variance N_0. By collecting received signal samples in a vector, the above model can be expressed in vectorial form as follows:

$$\mathbf{r} = \sum_{\ell=0}^{L-1} \text{diag}(\mathbf{s}_\ell)\, \mathbf{h}_\ell + \mathbf{w} \quad \in \mathcal{C}^N, \quad (3)$$

where $\mathbf{r}=[r(0), r(1), \cdots, r(N-1)]^T \in \mathcal{C}^N$, and $\mathbf{h}_\ell=[h(0,\ell), h(1,\ell), \cdots, h(N-1,\ell)]^T \in \mathcal{C}^N$, $\ell = 0, 1, \cdots, L-1$, represents L-path wide sense stationary uncorrelated scattering (WSSUS) Rayleigh fading coefficients. We assume Jake's channel model having exponentially decaying normalized multipath channel powers $\sigma_\ell^2 = e^{-\ell/L}/(\sum_{m=0}^{L-1} e^{-m/L})$, $\forall\, \ell$. Note that due to the cyclic prefix employed at the transmitter, $s(-\ell) = s(N-\ell)$ for $\ell = 0, 1, \cdots, L-1$. We now define

$$\begin{aligned} \mathbf{s}_\ell &= [\, s(-\ell), s(-(\ell-1)), \cdots, s(N-(\ell+1))\,]^T \in \mathcal{C}^N \\ &= \text{vshift}(\mathbf{s}, \ell) \;, \end{aligned} \quad (4)$$

where vshift$(\mathbf{s}, \ell)$ denotes the ℓ-step circular shift operator for a column vector $\mathbf{s} = [s(0),\, s(1),\, \cdots,\, s(N-1)]^T$. Defining

$$\mathbf{S}_\ell = \text{diag}(\mathbf{s}_\ell) \quad , \quad \mathbf{S}=[\mathbf{S}_0, \mathbf{S}_1, \cdots, \mathbf{S}_{L-1}] \in \mathcal{C}^{N\times LN} \quad (5)$$

and $\mathbf{h}=[\mathbf{h}_0^T, \mathbf{h}_1^T, \cdots, \mathbf{h}_{L-1}^T]^T \in \mathcal{C}^{LN}$, the receive signal model in (3) is rewritten as

$$\mathbf{r} = \mathbf{S}\mathbf{h} + \mathbf{w} \;. \quad (6)$$

III. DCT Expansions of the Multipath Channels

The number of unknown channel parameters to be estimated within one OFDM symbol interval is NL and it seems that the estimation of those coefficients is impossible even with pilot symbols since there are more unknowns to be determined than known equations. However, due to the banded character of the channel matrix in the frequency domain, it is possible to reduce the number of unknown channel parameters, substantially, by representing the channel by a suitable orthogonal series expansion and taking only significant expansion coefficients for estimation. To reduce number of unknowns from N to D, for each multipath, we employ the DCT for expansion of the ℓth multipath of the channel as

$$\mathbf{h}_\ell = \mathbf{\Psi}\mathbf{c}_\ell \;, \quad (7)$$

where $\mathbf{\Psi} \in \mathcal{C}^{N\times D}$ is the DCT matrix expressed as $\mathbf{\Psi} = [\boldsymbol{\psi}(0), \boldsymbol{\psi}(1) \cdots, \boldsymbol{\psi}(N-1)]^T$ and $\boldsymbol{\psi}(t) =$

$[\psi_0(t), \psi_1(t) \cdots, \psi_{D-1}(t)]^T$, where $\psi_d(t)$ is the DCT orthonal basis function. $\mathbf{c}_\ell \in \mathcal{C}^D$ is the coefficient vector for $\mathbf{h}_\ell$. Accordingly, the DCT expansion of the overall channel vector is given as

$$\mathbf{h} = \mathbf{\Phi c} \ , \tag{8}$$

where $\mathbf{c} = [\mathbf{c}_0^T, \mathbf{c}_1^T, \cdots, \mathbf{c}_{L-1}^T]^T \in \mathcal{C}^{LD}$, $\mathbf{\Phi} = \mathbf{I}_L \otimes \mathbf{\Psi} \in \mathcal{C}^{LN \times LD}$, $\otimes$ denotes the kronecker product and $\mathbf{I}_L$ is an $L \times L$ identity matrix. So, substituting (8) into (6) we have

$$\mathbf{r} = \mathbf{S\Phi c} + \mathbf{w} \ . \tag{9}$$

The dimension D of the basis expansion satisfies $\tilde{D} \leq D \leq N$. The lower bound is given by $\tilde{D} = [2(f_D)_{max} + 1]$, where $(f_D)_{max}$ is the maximum (one-sided) Doppler bandwidth defined by

$$(f_D)_{max} = \frac{v_{max} f_c}{c} T \ , \tag{10}$$

and v_{max}, f_c, c are the maximum supported velocity, the carrier frequency and the speed of light, respectively. T is the OFDM symbol duration.

IV. Channel Estimation Algorithm

The EM-MAP channel estimation algorithm is implemented in two steps. In the first step, called the expectation step (E-step), the auxiliary function

$$Q\big(\mathbf{c}|\mathbf{c}^{(i)}\big) = E_{\mathbf{s}}\big[\log p(\mathbf{r}|\mathbf{c},\mathbf{s}) \,\big|\, \mathbf{r}, \mathbf{c}^{(i)}\big] + \log p(\mathbf{c}) \tag{11}$$

is computed where $\mathbf{c}^{(i)}$ is the estimate of $\mathbf{c}$ at the ith iteration. The conditional expectation in (11) is taken with respect to $\mathbf{s}$ given the observation $\mathbf{r}$ and assumes that $\mathbf{c}$ equals its estimate calculated at iteration (i). Given the received signal $\mathbf{r}$, the EM algorithm starts with an initial value $\mathbf{c}^{(0)}$ of the unknown channel parameter $\mathbf{c}$, determined by the pilot symbols available. In the second step, called the maximization step (M-step), the unknown channel parameter vector $\mathbf{c}$ is updated according to

$$\mathbf{c}^{(i+1)} = \arg\max_{\mathbf{c}} Q\big(\mathbf{c}|\mathbf{c}^{(i)}\big) \tag{12}$$

After going through the mathematical details, the final form of the updating rule of the DCT coefficients (reduced dimensional channel coefficient vector) can be obtained as follows

$$\mathbf{c}^{(i+1)} = \mathbf{G}^{(i)^{-1}} \mathbf{F}^{(i)} \ , \tag{13}$$

where

$$\begin{aligned} \mathbf{G}^{(i)} &= \mathbf{\Sigma}_{\mathbf{c}}^{-1} + \frac{1}{N_0} \mathbf{\Phi}^\dagger \mathbf{A}^{(i)} \mathbf{\Phi} \quad \in \mathcal{C}^{LD \times LD}, \\ \mathbf{F}^{(i)} &= \mathbf{\Phi}^\dagger \mathbf{B}^{(i)^\dagger} \mathbf{r} \quad \in \mathcal{C}^{LD} \ , \end{aligned} \tag{14}$$

and $\mathbf{\Sigma}_{\mathbf{c}}$ is the covariance matrix of $\mathbf{c}$ which can be determined easily from the channel correlation matrix. After some algebra, we obtain the matrices

$$\begin{aligned} \mathbf{A}^{(i)} &= E_{\mathbf{s}}\big[\mathbf{S}^\dagger \mathbf{S} \big| \mathbf{r}, \mathbf{c}^{(i)}\big] \ \in \mathcal{C}^{LN \times LN} \\ &= \begin{bmatrix} \boldsymbol{\rho}_{0,0}^{(i)} & \cdots & \boldsymbol{\rho}_{0,L-1}^{(i)} \\ \vdots & \ddots & \vdots \\ \boldsymbol{\rho}_{L-1,0}^{(i)} & \cdots & \boldsymbol{\rho}_{L-1,L-1}^{(i)} \end{bmatrix} \end{aligned} \tag{15}$$

and

$$\begin{aligned} \mathbf{B}^{(i)} &= E_{\mathbf{s}}\big[\mathbf{S} \big| \mathbf{r}, \mathbf{c}^{(i)}\big] \ \in \mathcal{C}^{N \times LN} \\ &= \Big[\text{diag}\Big(\text{vshift}\big(\boldsymbol{\mu}_{\mathbf{s}}^{(i)}, 0\big)\Big), \text{diag}\Big(\text{vshift}\big(\boldsymbol{\mu}_{\mathbf{s}}^{(i)}, 1\big)\Big), \\ &\quad \cdots, \text{diag}\Big(\text{vshift}\big(\boldsymbol{\mu}_{\mathbf{s}}^{(i)}, L-1\big)\Big)\Big] \end{aligned} \tag{16}$$

in (14), where $\boldsymbol{\mu}_{\mathbf{s}}^{(i)} = E_{\mathbf{s}}\big[\mathbf{s} \big| \mathbf{r}, \mathbf{c}^{(i)}\big] \in \mathcal{C}^N$ and $\boldsymbol{\rho}_{p,q}^{(i)} = E_{\mathbf{s}}\big[\mathbf{S}_p^\dagger \mathbf{S}_q \big| \mathbf{r}, \mathbf{c}^{(i)}\big]$. In order to obtain $\boldsymbol{\mu}_{\mathbf{s}}^{(i)}$, the average of the transmitted signal in the time-domain, we consider the following alternative form of the observation equation in (6):

$$\mathbf{r} = \mathbf{H s} + \mathbf{w} \ . \tag{17}$$

It is straightforward to see that $\mathbf{H}$ stands for the convolution matrix and can be expressed as

$$\mathbf{H} = \sum_{\ell=0}^{L-1} \text{mshift}\big(\text{diag}(\mathbf{h}_\ell), 0, -\ell\big), \tag{18}$$

where mshift$(\mathbf{A}, q, p)$ represents row-wise q-step and column-wise p-step circular shift of matrix $\mathbf{A}$. Consequently, we obtain the posterior mean of $\mathbf{s}$ given $\mathbf{c}^{(i)}$ at ith step as follows:

$$\boldsymbol{\mu}_{\mathbf{s}}^{(i)} = \mathbf{s}_P + \frac{1}{N_0} \mathbf{\Sigma}_{\mathbf{s}}^{(i)} \mathbf{H}^{(i)^\dagger} \big(\mathbf{r} - \mathbf{H}^{(i)} \mathbf{s}_P\big) \ , \tag{19}$$

together with the following posterior covariance matrix of $\mathbf{s}$:

$$\mathbf{\Sigma}_{\mathbf{s}}^{(i)} = \mathbf{\Xi}_{\mathbf{s}} \Big(\mathbf{I}_N + \frac{1}{N_0} \mathbf{H}^{(i)^\dagger} \mathbf{H}^{(i)} \mathbf{\Xi}_{\mathbf{s}}\Big)^{-1} \ , \tag{20}$$

where $\mathbf{\Xi}_{\mathbf{s}} = E_{\mathbf{s}}\big[(\mathbf{s} - \mathbf{s}_P)(\mathbf{s} - \mathbf{s}_P)^\dagger\big]$ and $\mathbf{H}^{(i)}$ is given by (18). Note that, in (19), $\mathbf{s}_P = \mathbb{F}^{-1} \mathbf{d}_P$ and $\mathbf{d}_P$ is the frequency domain pilot symbol vector with the following entries:

$$d_P(q) = \begin{cases} d(q) & , \ q \in \{0, \Delta, 2\Delta, \cdots, (P-1)\Delta\} \\ 0 & , \ \text{otherwise} \end{cases} \quad , \ q = 0, 1, \cdots, K-1 \ . \tag{21}$$

Δ and P in (21) denote the pilot spacing and the number of pilots in one OFDM block, respectively. Subsequently, in (15), we obtain

$$\boldsymbol{\rho}_{p,q}^{(i)} = \text{diag}\Big(\text{dg}\big(\text{mshift}(\mathbf{R}_{\mathbf{s}}^{(i)}, q, p)\big)\Big) \ , \tag{22}$$

where the operator dg$(\cdot)$ denotes the main diagonal vector of a matrix, and $\mathbf{R}_{\mathbf{s}}^{(i)}$ represents the posterior autocorrelation matrix of $\mathbf{s}$ given $\mathbf{c}^{(i)}$. The latter quantity is given by

$$\mathbf{R}_{\mathbf{s}}^{(i)} = \boldsymbol{\mu}_{\mathbf{s}}^{(i)} \boldsymbol{\mu}_{\mathbf{s}}^{(i)^\dagger} + \mathbf{\Sigma}_{\mathbf{s}}^{(i)} \ . \tag{23}$$

A. Initialization of the EM-MAP Algorithm

Using the inverse Fourier transform, the following equality is obtained for the diagonal matrix $\mathbf{S}_\ell$ in (5):

$$\begin{aligned} \mathbf{S}_\ell &= \text{diag}(\text{vshift}(\mathbf{s}, \ell)) \\ &= \frac{1}{N} \sum_{q=0}^{K-1} d(q) \, e^{-j2\pi \ell q/N} \, \text{diag}\big(\mathbb{F}_N^*(q)\big) \ , \end{aligned} \tag{24}$$

where $(\cdot)^*$ stands for the complex conjugate operation and $\mathbb{F}_N(q)$ denotes the qth column of the DFT matrix. Substituting

$\mathbf{Z} = \mathbf{S}\mathbf{\Phi}$ in (9) we have the following alternative form of the receive signal model:

$$\mathbf{r} = \mathbf{Z}\,\mathbf{c} + \mathbf{w}. \tag{25}$$

Using (24), it is straightforward that

$$\begin{aligned}\mathbf{Z} &= \mathbf{S}\mathbf{\Phi} = [\mathbf{S}_0\boldsymbol{\psi}, \mathbf{S}_1\boldsymbol{\psi}, \cdots, \mathbf{S}_{L-1}\boldsymbol{\psi}] \\ &= \sum_{q=0}^{K-1} d(q)\mathbf{U}_q\ ,\end{aligned} \tag{26}$$

with

$$\mathbf{U}_q = \mathbb{F}_L^T(q) \otimes \left(\left(\mathbf{1}_D^T \otimes \frac{1}{N}\,\mathbb{F}_N^*(q)\right) \odot \mathbf{\Psi}\right)\ , \tag{27}$$

where $\mathbb{F}_L(q)$ represents the first L terms of the qth column of the DFT matrix $\mathbb{F}$, $\odot$ denotes the element by element product and $\mathbf{1}_D$ stands for the all-one column vector with length D. In (26), we consider $\mathbf{Z} = \mathbf{Z}_P + \mathbf{Z}_D$, where $\mathbf{Z}_P = \sum_{q\in\mathcal{I}_P}^{K-1} d(q)\mathbf{U}_q$ and $\mathbf{Z}_D = \sum_{q\in\mathcal{I}_D}^{K-1} d(q)\mathbf{U}_q$ are the matrices obtained from pilot and data symbols, respectively. So, the initial value of the reduced dimensional channel vector $\mathbf{c}$ can be determined from the received signal model (25) by a linear minimum mean square error (MMSE) estimation technique as follows:

$$\mathbf{c}^{(0)} = \mathbf{\Sigma_c}\,\mathbf{Z}_P^\dagger\Big(N_0\mathbf{I}_N + \mathbf{Z}_P\mathbf{\Sigma_c}\mathbf{Z}_P^\dagger + \sum_{q\in\mathcal{I}_D}\mathbf{U}_q\mathbf{\Sigma_c}\mathbf{U}_q^\dagger\Big)^{-1}\mathbf{r}\,. \tag{28}$$

V. Complexity Analysis

The computational complexity of the algorithm is presented in Table-I under the assumption that $K = N$. Note that,

TABLE I: Computational Complexity Details

Eq. No	Variable	Complexity (CMs)
(28)	$\mathbf{c}^{(0)}$	NLD
(18) (using (7))	$\mathbf{H}^{(i)}$	NDL^2
(19)	$\boldsymbol{\mu}_\mathbf{s}^{(i)}$	$NL + \Delta L$
(20)	$\mathbf{\Sigma}_\mathbf{s}^{(i)}$	$2\Delta^2 + 2\Delta L$
(23)	$\mathbf{R}_\mathbf{s}^{(i)}$	$N(N+1)/2$
(15) and (16)	$\mathbf{A}^{(i)}$ and $\mathbf{B}^{(i)}$	0
(14)	$\mathbf{G}^{(i)}$ $\mathbf{F}^{(i)}$	$ND^2L(L+1)/2$ $ND(L+1)$
(13)	$\mathbf{c}^{(i+1)}$	$2D^2L^2$

in the initialization step of the algorithm in (28), the term $\mathbf{\Sigma_c}\,\mathbf{Z}_P^\dagger\big(\mathbf{Z}_P\mathbf{\Sigma_c}\mathbf{Z}_P^\dagger + \sum_{q\in\mathcal{I}_D}\mathbf{U}_q\mathbf{\Sigma_c}\mathbf{U}_q^\dagger + N_0\mathbf{I}_N\big)^{-1} \in \mathcal{C}^{DL\times N}$ is a precomputed matrix. Therefore, the initialization step requires only a multiplication of this precomputed matrix with the $N\times 1$ $\mathbf{r}$ vector resulting in DLN complex multiplications (CMs). On the other hand, the covariance matrix $\mathbf{\Xi}_s$, necessary for computation of (19) and (20), is a block matrix whose submatrices are diagonal with constant entries. Also, the convolution matrix $\mathbf{H}^{(i)}$ in (19) and (20) is a sparse matrix whose columns have only L non-zero entries. Consequently, in the computation of $\boldsymbol{\mu}_\mathbf{s}^{(i)}$ and $\mathbf{\Sigma}_\mathbf{s}^{(i)}$ in (19) and (20), the terms $\mathbf{\Xi_s}\,\mathbf{H}^{(i)\dagger}$, $\mathbf{H}^{(i)\dagger}\mathbf{H}^{(i)}\mathbf{\Xi_s}$ and $\big(\mathbf{I}_N + \frac{1}{N_0}\mathbf{H}^{(i)\dagger}\mathbf{H}^{(i)}\mathbf{\Xi_s}\big)^{-1}$ can be approximated by block matrices whose submatrices are diagonal with constant entries resulting in a reduced complexity algorithm. As a result, it follows from Table-I that the total computational complexity per detected symbol of the channel estimation algorithm presented in this work is approximately $(N+1)/2 + D^2L^2/2 + DL^2 + 2DL + L \;\sim\; \mathcal{O}(N)$.

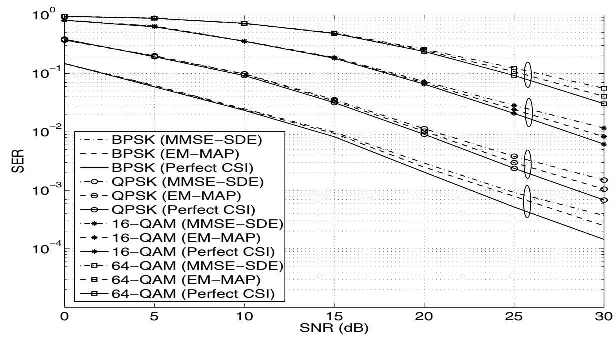
VI. Simulation Results

In this section, we present computer simulation results to assess the performance of the OFDM systems operating with the proposed channel estimation algorithm. Simulation parameters are chosen as in Table-II. The initial estimate of

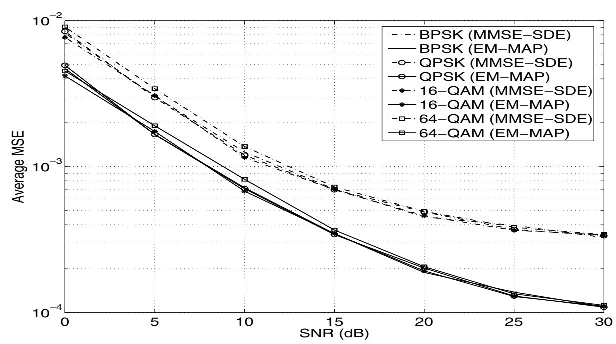
TABLE II: Simulation Parameters

Bandwidth (BW)	10 MHz
Carrier Frequency (f_c)	2.5 GHz
Number of Subcarriers (N)	1024
Number of Multipaths (L)	3
Number of DCT Coefficients (D)	3, 5
Number of Iterations (i_{max})	5
Pilot Spacing (Δ)	8, 12
Modulation Formats	BPSK, QPSK, 16-QAM, 64-QAM

the channel is performed by the reduced-complexity linear MMSE estimation techniques based on the pilot symbols. We refer to this method for obtaining the initial channel and data estimates as the MMSE separate detection and estimation (MMSE-SDE) scheme. The solid and the dashed curves in



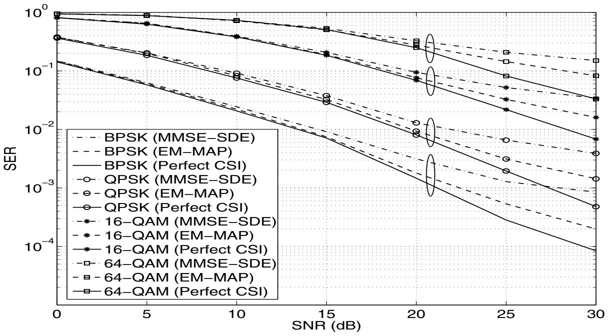
(a) SER vs. SNR



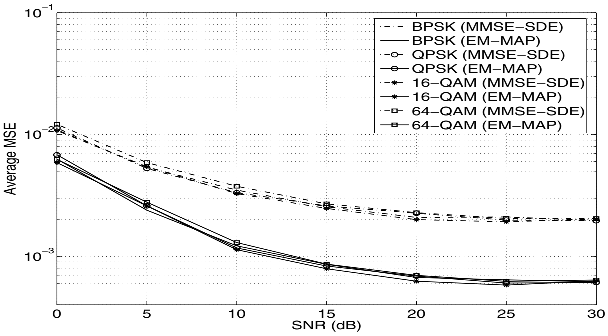
(b) Average MSE vs. SNR

Fig. 1: SER and MSE performance of the EM-MAP and MMSE-SDE algorithms for $f_DT = 0.0284$ ($v = 120$ km/h), $N = K = 1024$, $\Delta = 8$, $L = 3$, $D = 3$

Figures 1 and 2 represent the SER and mean-square error (MSE) performance curves of the EM-MAP and MMSE-SDE algorithms, when the pilot spacing is chosen as $\Delta = 8$ and the corresponding mobilities are $f_DT = 0.0284$ ($v = 120$ km/h) and $f_DT = 0.0852$ ($v = 360$ km/h). The multipath wireless channel having an exponentially decaying power delay profile



(a) SER vs. SNR



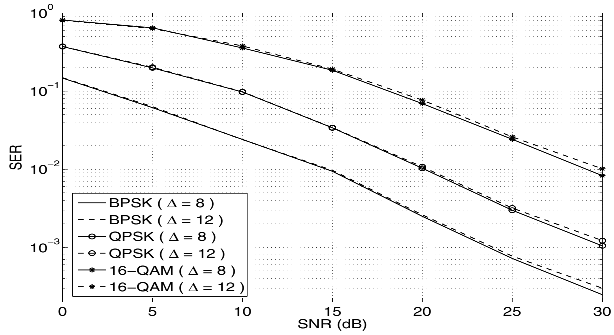
(b) Average MSE vs. SNR

Fig. 2: SER and MSE performance of the EM-MAP and MMSE-SDE algorithms for $f_D T = 0.0852$ ($v = 360$ km/h), $N = K = 1024$, $\Delta = 8$, $L = 3$, $D = 5$

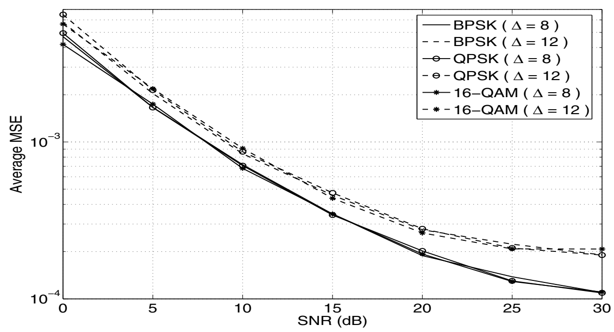
with the normalized powers, $\sigma_0^2 = 0.448$, $\sigma_1^2 = 0.321$, and $\sigma_2^2 = 0.230$, is chosen. It is observed that a maximum of three iterations are sufficient in order for the EM-MAP algorithm to converge. We conclude from these curves that even when the number of DCT coefficients is chosen to be fairly small as compared to the total number of coefficients, the performance loss in SER is not significant when channel state information (CSI) not available. We also observe that the SER performance of the EM-MAP algorithm obtained at the end of the third iteration step is better than that of the MMSE-SDE and the performance difference becomes more significant at higher mobilities. On the other hand, we observe that the average MSE performance of the EM-MAP algorithm is substantially better than that of the MMSE-SDE. In Fig. 3, effects of channel estimation on the average MSE and on the SER performance are investigated as functions of the pilot spacing (Δ) with the mobility $f_D T = 0.0284$ ($v = 120$ km/h). It is concluded from Fig. 3 that the SER and MSE performances do not change significantly for pilot spacings equal to 8 and 12.

VII. Conclusions

In this work, the problem of iterative channel estimation has been investigated and a new iterative channel estimation algorithm has been proposed for OFDM systems operating over frequency selective and very rapidly time-varying channels. The channel estimation algorithm is based on the EM-MAP technique which incorporates also the channel equalization and the data detection. The band-limited cosine orthogonal basis functions have been employed to describe the rapidly time-varying channel. Initial channel coefficients are effectively obtained by the pilot aided MMSE estimator and unknown data symbols are averaged out in the algorithm. It has been shown via computer simulation that the proposed algorithm has excellent symbol error rate and channel estimation performance even with a very small number of channel expansion coefficients, resulting in reduction of the computational complexity substantially.



(a) SER vs. SNR



(b) Average MSE vs. SNR

Fig. 3: SER and MSE performance of the EM-MAP algorithm with different pilot spacing for $f_D T = 0.0284$ ($v = 120$ km/h), $N = K = 1024$, $L = 3$, $D = 3$

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