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# Bianchi surfaces whose asymptotic lines are geodesic parallels

**Abstract:** It is proved that every Bianchi surface in $E^3$ of class $C^4$ whose asymptotic lines are geodesic parallels is either a helicoid or a surface of revolution.

**Keywords:** Bianchi surface, asymptotic line, geodesic parallel, geodesic ellipse, geodesic hyperbola, helicoidal surface.

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## 1 Introduction

Let $S$ be a smooth surface in the euclidean space $E^3$ with negative Gaussian curvature $K = -1/\rho^2$ where $\rho > 0$. If the asymptotic lines on $S$ are taken as the parametric lines, the fundamental forms become

$$\mathrm{I} = Edu^2 + 2Fdudv + Gdv^2, \tag{1}$$

$$\mathrm{II} = 2Mdudv, \tag{2}$$

where $u$ and $v$ are the asymptotic parameters. $S$ is called a *Bianchi surface* if $K$ can be expressed in the asymptotic parameters $(u, v)$ as

$$K = -1/\rho^2, \quad \rho = U(u) + V(v) \tag{3}$$

where $U(u)$ and $V(v)$ are arbitrary functions of their arguments [1; 2; 3]. Equivalently, a Bianchi surface can be characterized by

$$\frac{\partial^2}{\partial u \partial v}(-K)^{-1/2} = 0. \tag{4}$$

Bianchi surfaces have been studied by a number of mathematicians and physicists [4; 6; 5; 7; 8] from the view point of integrable systems. In [4], A. Fujioka introduced the concept of a generalized Chebyshev net and then proved that a Bianchi surface with constant Chebyshev angle parametrized by a generalized Chebyshev net is a piece of a right helicoid. It is well-known that the asymptotic line nets on a hyperbolic surface in $E^3$ have some important properties. Namely, if the asymptotic lines form a Chebyshev net on such surfaces, then these surfaces are of constant Gaussian curvature which constitute a special class of Bianchi surfaces.

In the present paper, Bianchi surfaces whose asymptotic lines constitute a system of geodesic parallels are considered and it is proved that such surfaces are helicoids or surfaces of revolution. It is to be noted that the net of asymptotic lines on $S$ which are geodesic parallels constitutes an example for generalized Chebyshev nets with non-constant Chebyshev angle. In what follows we will assume that $\omega$ is not constant since otherwise $K$ would be zero. We need the following theorem:

**Theorem** ([3; 9]). *If two independent systems of geodesic parallels on S are chosen as parametric lines, then the element of arc can be written in the form*

$$ds^2 = \csc^2 \omega (du^2 + 2\cos\omega\, dudv + dv^2), \tag{5}$$

*where $\omega$ with $0 < \omega < \pi$ is the angle between the parametric lines and conversely.*

From (1), (2) and (5) it follows that

$$E = G = \csc^2\omega, \quad F = \cos\omega\csc^2\omega \tag{6}$$

$$L = N = 0, \quad M \neq 0 \tag{7}$$

$$K = -\left(\frac{M}{H}\right)^2, \quad H = \sqrt{EG - F^2}. \tag{8}$$

Using (6), (7) and (8), we can write the Mainardi–Codazzi equations ([3, p. 156], [9, p. 111]) in the form

$$-\frac{\partial}{\partial u}\left(\frac{M}{H}\right) - 2\Gamma_{12}^2\left(\frac{M}{H}\right) = 0, \qquad -\frac{\partial}{\partial v}\left(\frac{M}{H}\right) - 2\Gamma_{12}^1\left(\frac{M}{H}\right) = 0$$

or

$$(\ln\sqrt{-K})_u = -2\Gamma_{12}^2, \tag{9}$$

$$(\ln\sqrt{-K})_v = -2\Gamma_{12}^1 \tag{10}$$

with $\Gamma_{12}^1 = \frac{1}{2}H^{-2}(EE_v - FE_u)$ and $\Gamma_{12}^2 = \frac{1}{2}H^{-2}(EE_u - FE_v)$, see [9]. The integrability condition for (9) and (10) is

$$(\Gamma_{12}^1)_u = (\Gamma_{12}^2)_v. \tag{11}$$

By (6), the Equations (9), (10) and (11) take the respective forms

$$(\ln\sqrt{-K})_u = 2\cot\omega\csc^2\omega(\omega_u - \omega_v\cos\omega), \tag{12}$$

$$(\ln\sqrt{-K})_v = 2\cot\omega\csc^2\omega(\omega_v - \omega_u\cos\omega), \tag{13}$$

$$(2 + \cos^2\omega)(\omega_u^2 - \omega_v^2) - (\omega_{uu} - \omega_{vv})\sin\omega\cos\omega = 0. \tag{14}$$

On the other hand, by (6), the Gaussian curvature ([9, p. 114]) of $S$ is found as

$$K = -({\omega_u}^2 + {\omega_v}^2)\frac{1 + \cos^2\omega}{\sin^2\omega} + (\omega_{uu} + \omega_{vv})\frac{\cos\omega}{\sin\omega} + 4\frac{\cos\omega}{\sin^2\omega}\omega_u\omega_v - \frac{1 + \cos^2\omega}{\sin\omega}\omega_{uv}. \tag{15}$$

We now introduce the new parameters $(\xi, \eta)$ defined by

$$u + v = \xi, \quad u - v = \eta.$$

We note that the curves $\xi = \text{const}$ and $\eta = \text{const}$ are respectively, the geodesic ellipses and geodesic hyperbolas [3; 9]. Then, since

$$\omega_u = \omega_\xi + \omega_\eta, \quad \omega_v = \omega_\xi - \omega_\eta, \quad \omega_{uu} = \omega_{\xi\xi} + 2\omega_{\xi\eta} + \omega_{\eta\eta}, \quad \omega_{vv} = \omega_{\xi\xi} - 2\omega_{\xi\eta} + \omega_{\eta\eta}, \quad \omega_{uv} = \omega_{\xi\xi} - \omega_{\eta\eta},$$

equations (12), (13) and (14) transform respectively into

$$(\ln\sqrt{-K})_\xi + (\ln\sqrt{-K})_\eta = 2\cot\omega\csc^2\omega[(1 - \cos\omega)\omega_\xi + (1 + \cos\omega)\omega_\eta], \tag{16}$$

$$(\ln\sqrt{-K})_\xi - (\ln\sqrt{-K})_\eta = 2\cot\omega\csc^2\omega[(1 - \cos\omega)\omega_\xi - (1 + \cos\omega)\omega_\eta], \tag{17}$$

$$\omega_{\xi\eta} = \frac{2 + \cos^2\omega}{\sin\omega\cos\omega}\omega_\xi\omega_\eta \quad \text{with } \omega \neq \frac{\pi}{2}. \tag{18}$$

Adding and subtracting (16) and (17) side by side we obtain

$$(\ln\sqrt{-K})_\xi = 2\cot\omega\csc^2\omega(1 - \cos\omega)\omega_\xi, \tag{19}$$

$$(\ln\sqrt{-K})_\eta = 2\cot\omega\csc^2\omega(1 + \cos\omega)\omega_\eta. \tag{20}$$

Integration of (19) and (20) gives respectively

$$\sqrt{-K} = r(\eta)\tan\frac{\omega}{2}e^{-\frac{1}{2}\tan^2\frac{\omega}{2}}, \tag{21}$$

$$\sqrt{-K} = s(\xi)\cot\frac{\omega}{2}e^{-\frac{1}{2}\cot^2\frac{\omega}{2}}, \tag{22}$$

where $r(\eta)$ and $s(\xi)$ are arbitrary positive functions of their arguments. From (21) and (22) we get

$$t = a(\xi) + b(\eta) = \frac{1}{2}\left(\tan^2\frac{\omega}{2} - \cot^2\frac{\omega}{2}\right) - 2\ln\left(\tan\frac{\omega}{2}\right) \tag{23}$$

where $a(\xi) = -\ln s(\xi)$ and $b(\eta) = \ln r(\eta)$. According to the implicit function theorem, under certain Conditions (23) defines $\omega$ as a function of $a(\xi) + b(\eta)$ which will be denoted by

$$\omega = \omega(t). \tag{24}$$

Differentiating (23) with respect to $t$ we find that

$$\begin{aligned} \omega'(t) &= \frac{1}{4}\tan^2\omega\sin\omega, \\ \omega''(t) &= \frac{1}{16}\tan^5\omega(2+\cos^2\omega) \end{aligned} \tag{25}$$

which will be needed later. On the other hand, $\omega(t)$ must satisfy the integrability Condition (18). Since

$$\omega_\xi = \omega'(t)\,a'(\xi), \quad \omega_\eta = \omega'(t)\,b'(\eta), \quad \omega_{\xi\eta} = \omega''(t)\,a'(\xi)\,b'(\eta)$$

equation (18) is transformed into

$$\left[\omega''(t) - \frac{2+\cos^2\omega(t)}{\cos\omega(t)\sin\omega(t)}\,\omega'^2(t)\right]a'(\xi)b'(\eta) = 0$$

in which primes indicate the derivatives with respect to the corresponding variables.

Here we distinguish three cases:

**Case 1.** $a'(\xi) = b'(\eta) = 0$. This implies $\omega = \omega(t) = \text{const}$. This cannot happen since $K \neq 0$.

**Case 2.** $a'(\xi) \neq 0, b'(\eta) = 0$ or $a'(\xi) = 0, b'(\eta) \neq 0$. In this case $\omega$ and, consequently, by (6), (8) and (22), the coefficients of the fundamental forms of $S$ depend on the single parameter $\xi = u + v$. But this means that $S$ is a helicoid or a surface of revolution [3].

**Case 3.** $a'(\xi).b'(\eta) \neq 0$ and $\omega''(t) - \frac{2+\cos^2\omega(t)}{\cos\omega(t)\sin\omega(t)}\,\omega'^2(t) = 0$. The general solution of this differential equation is found to be

$$\begin{aligned} \frac{1}{8}\left(\tan^2\frac{\omega}{2} - \cot^2\frac{\omega}{2}\right) - \frac{1}{2}\ln\left(\tan\frac{\omega}{2}\right) &= ct + c_1, \\ t &= a(\xi) + b(\eta) \end{aligned} \tag{26}$$

where $c_1$ and $c > 0$ are arbitrary constants. Comparing (23) with (26) we obtain

$$\left(c - \frac{1}{4}\right)[a(\xi) + b(\eta)] + c_1 = 0. \tag{27}$$

If $c \neq \frac{1}{4}$, equation (27) implies that $a(\xi) = \text{const}$, $b(\eta) = \text{const}$ which contradicts the first hypothesis $a'.b' \neq 0$ in the Case 3. So we have $c = \frac{1}{4}$ and $c_1 = 0$. Then (26) reduces to

$$\begin{aligned} t &= a(\xi) + b(\eta) \\ &= \frac{1}{2}\left(\tan^2\frac{\omega}{2} - \cot^2\frac{\omega}{2}\right) - 2\ln\left(\tan\frac{\omega}{2}\right), \quad a'.b' \neq 0. \end{aligned} \tag{28}$$

Hence we have

**Theorem 1.** *If the two families of the asymptotic lines on S are geodesic parallels, then S is either a helicoid or a surface of revolution, or the angle $\omega$ between the asymptotic lines which allows us to determine S is given by*

$$\frac{1}{2}\left(\tan^2\frac{\omega}{2} - \cot^2\frac{\omega}{2}\right) - 2\ln\left(\tan\frac{\omega}{2}\right) = a(\xi) + b(\eta), \quad a'.b' \neq 0.$$

## 2 Bianchi surfaces whose asymptotic lines constitute a system of geodesic parallels

Since, under the transformation $\xi = u + v, \eta = u - v$,

$$\begin{aligned}
\omega_u &= \omega'(t)(a'(\xi) + b'(\eta)),\\
\omega_v &= \omega'(t)(a'(\xi) - b'(\eta)),\\
\omega_{uv} &= \omega''(t)(a'^2(\xi) - b'^2(\eta)) + \omega'(t)(a''(\xi) - b''(\eta)),\\
\omega_{uu} &= \omega''(t)(a'(\xi) + b'(\eta))^2 + \omega'(t)(a''(\xi) + b''(\eta)),\\
\omega_{vv} &= \omega''(t)(a'(\xi) - b'(\eta))^2 + \omega'(t)(a''(\xi) + b''(\eta)),
\end{aligned}$$

Equation (15) transforms into

$$\begin{aligned}
K = \big[-(1-\cos\omega)^2 a'^2 + (1+\cos\omega)^2 b'^2\big]\frac{\omega''}{\sin\omega} - \big[(1-\cos\omega)^2 a'^2 + (1+\cos\omega)^2 b'^2\big]\frac{2\omega'^2}{\sin^2\omega}\\
+ \big[-(1-\cos\omega)^2 a'' + (1+\cos\omega)^2 b''\big]\frac{\omega'}{\sin\omega}.
\end{aligned} \tag{29}$$

Using (22), (25) and (29), we obtain

$$\begin{aligned}
(1-\cos\omega)^2 a'' + \frac{1}{4}\tan^2\omega\sec\omega\big[\sin^4\omega + (1-\cos\omega)^2\big]\, a'^2\\
- (1+\cos\omega)^2\, b'' - \frac{1}{4}\tan^2\omega\sec\omega\big[\sin^4\omega + (1+\cos\omega)^2\big]\, b'^2 = 4\cot^2\frac{\omega}{2}\cot^2\omega\, e^{-2a(\xi)} e^{-\cot^2\frac{\omega}{2}}
\end{aligned}$$

or

$$a'' - \cot^4\frac{\omega}{2}\, b'' + \frac{\tan^2\omega}{4\cos\omega}\left(4\cos^4\frac{\omega}{2} + 1\right)a'^2 - \frac{\tan^2\omega}{4\cos\omega}\cot^4\frac{\omega}{2}\left(4\sin^4\frac{\omega}{2} + 1\right)b'^2 = \frac{\cot^2\frac{\omega}{2}}{\sin^4\frac{\omega}{2}}\cot^2\omega\, e^{-2a(\xi)}\, e^{-\cot^2\frac{\omega}{2}}. \tag{30}$$

On the other hand, by (3) and (22) we find

$$\rho = \frac{1}{\sqrt{-K}} = e^{a(\xi)}\tan\frac{\omega}{2}\, e^{\frac{1}{2}\cot^2\frac{\omega}{2}}. \tag{31}$$

Suppose now that $S$ is a Bianchi surface. According to (4), the condition for $S$ to be a Bianchi surface is

$$\begin{aligned}
\frac{\partial^2}{\partial u\partial v}(-K)^{-1/2} = -4e^{a(\xi)}\, e^{\frac{1}{2}\cot^2\frac{\omega}{2}}\frac{\sin^4\frac{\omega}{2}}{\sin 2\omega}\Big\{a'' - \cot^2\frac{\omega}{2}\, b'' + \Big[1 + \cot^2\frac{\omega}{2} + \frac{1}{4}\cot^2\frac{\omega}{2}\tan^2\omega\Big(\sec\omega - \cot^2\frac{\omega}{2}\Big)\Big]a'^2\\
- \Big[\frac{1}{4}\cot^2\frac{\omega}{2}\tan^2\omega\Big(\sec\omega - \cot^2\frac{\omega}{2}\Big)\Big]b'^2\Big\} = 0
\end{aligned}$$

from which it follows that

$$a'' - \cot^2\frac{\omega}{2}\, b'' + \Big[1 + \cot^2\frac{\omega}{2} + \frac{1}{4}\cot^2\frac{\omega}{2}\tan^2\omega\Big(\sec\omega - \cot^2\frac{\omega}{2}\Big)\Big]a'^2 - \Big[\frac{1}{4}\cot^2\frac{\omega}{2}\tan^2\omega\Big(\sec\omega - \cot^2\frac{\omega}{2}\Big)\Big]b'^2 = 0. \tag{32}$$

Since

$$\det\begin{bmatrix} 1 & -\cot^4\frac{\omega}{2} \\ 1 & -\cot^2\frac{\omega}{2}\end{bmatrix} = \cot^2\frac{\omega}{2}\csc^2\frac{\omega}{2}\cos\omega \neq 0,$$

(30) and (32) can be solved for $a''$ and $b''$. Calculations being done (which are also verified by using a symbolic computation package) we obtain

$$a'' = A_1 a'^2 + B_1 b'^2 - C_1 e^{-2a}, \tag{33}$$

$$b'' = A_2 a'^2 + B_2 b'^2 - C_2 e^{-2a}, \tag{34}$$

where

$$\begin{aligned}
A_1 &= -\frac{1}{32}(-4 + 11\cos\omega + \cos 3\omega)\sec^2\omega\tan^2\omega,\\
B_1 &= \frac{1}{8}\cos^4\frac{\omega}{2}(-12 + 11\cos\omega - 8\cos 2\omega + \cos 3\omega)\cot^2\frac{\omega}{2}\sec^4\omega,\\
C_1 &= \frac{1}{4}e^{-\cot^2\frac{\omega}{2}}\cos\omega\csc^6\frac{\omega}{2},\\
A_2 &= \frac{1}{8}(12 + 11\cos\omega + 8\cos 2\omega + \cos 3\omega)\sec^4\omega\sin^4\frac{\omega}{2}\tan^2\frac{\omega}{2},\\
B_2 &= -\frac{1}{32}(4 + 11\cos\omega + \cos 3\omega)\sec^2\omega\tan^2\omega,\\
C_2 &= e^{-\cot^2\frac{\omega}{2}}\cot\omega\csc^2\frac{\omega}{2}\csc\omega.
\end{aligned}$$

Differentiating (33) with respect to $\eta$ and using the fact that $\omega' b' \neq 0$ we find

$$a'^2\frac{dA_1}{d\omega} + \frac{2b'' B_1}{\omega'} + b'^2\frac{dB_1}{d\omega} - e^{-2a(\xi)}\frac{dC_1}{d\omega} = 0. \tag{35}$$

If we use (34), equation (35) becomes

$$A_3 a'^2 + B_3 b'^2 = C_3 e^{-2a}, \tag{36}$$

where

$$A_3 = \frac{dA_1}{d\omega} + \frac{2B_1A_2}{\omega'},\quad B_3 = \frac{2B_1B_2}{\omega'} + \frac{dB_1}{d\omega},\quad C_3 = \frac{2B_1C_2}{\omega'} + \frac{dC_1}{d\omega}.$$

Similarly, differentiating (34) with respect to $\xi$ and remembering that $\omega' a' \neq 0$ and using (33) we have

$$A_4 a'^2 + B_4 b'^2 = C_4 e^{-2a}, \tag{37}$$

where

$$A_4 = \frac{2A_1A_2}{\omega'} + \frac{dA_2}{d\omega},\quad B_4 = \frac{2A_2B_1}{\omega'} + \frac{dB_2}{d\omega},\quad C_4 = \frac{dC_2}{d\omega} - \frac{2C_2}{\omega'} + \frac{2A_2C_1}{\omega'}.$$

Since

$$\det\begin{bmatrix} A_3 & B_3 \\ A_4 & B_4 \end{bmatrix} = \frac{1}{4}(5 + \cos 2\omega)\sec^6\omega\tan^2\omega \neq 0,$$

the system defined by (36) and (37) can be solved for $a'^2$ and $b'^2$ yielding

$$a'^2 = \lambda(a,\omega), \tag{38}$$

$$b'^2 = \mu(a,\omega), \tag{39}$$

where

$$\begin{aligned}
\lambda(a,\omega) &= \frac{1}{16}e^{-2a}e^{-\cot^2\frac{\omega}{2}}\cos^2\omega(5 + 3\cos 2\omega)\csc^8\frac{\omega}{2},\\
\mu(a,\omega) &= e^{-2a}e^{-\cot^2\frac{\omega}{2}}(5 + 3\cos 2\omega)\cot^2\omega\csc^2\omega.
\end{aligned}$$

From (38) it follows that $\omega$ is a function of $\xi$. Then (28) implies that $b(\eta) = \text{const}$ which contradicts the hypothesis $a'(\xi)b'(\xi) \neq 0$ involved in Case 3. Therefore, Case 3 cannot happen. So, only Case 2 should be considered which means that $S$ is a helicoid or a surface of revolution [3].

On the other hand, according to Bour's theorem ([3, p. 147]) every helicoid is applicable to some surface of revolution.

Now it remains to determine $\omega$ in terms of $a(\xi)$. Then all the coefficients of the fundamental forms of $S$ can readily be obtained. To this end, in Case 2, let $a'(\xi) \neq 0$, $b'(\eta) = 0$. Then, (33) and (34) take the respective forms

$$a'' = A_1 a'^2 - C_1 e^{-2a}, \tag{40}$$

$$0 = A_2 a'^2 - C_2 e^{-2a}. \tag{41}$$

Eliminating $a'^2$ between (40) and (41) and observing that $A_2 \neq 0$ for $\omega \in (0, \pi)$, we obtain

$$e^{2a} a'' = \frac{A_1 C_2 - A_2 C_1}{A_2} = \Lambda(a) \tag{42}$$

where

$$\Lambda(a) = \frac{-16 e^{-\cot^2 \frac{\omega}{2}} \csc^2 2\omega (2\cos\omega + \sin^2\omega) \cos^4\omega}{(12 + 11\cos\omega + 8\cos 2\omega + \cos 3\omega) \tan^2 \frac{\omega}{2} \sin^4 \frac{\omega}{2}} \quad \text{and} \quad \omega = \omega(a).$$

Putting $a' = z, a'' = \frac{dz}{da} a' = z \frac{dz}{da}$ in (42) and remembering that $a' \neq 0$, we get

$$z\, dz = e^{-2a} \Lambda(a)\, da,$$

the integration of which gives

$$\frac{z^2}{2} = \frac{a'^2}{2} = \int e^{-2a} \Lambda(a)\, da + c_0$$

with an arbitrary constant $c_0$, or

$$\int \frac{da}{\Omega(a)} = \xi + c_2 \tag{43}$$

with an arbitrary constant $c_2$ and

$$\Omega(a) = \mp \sqrt{2\Big[\int e^{-2a} \Lambda(a)\, da + c_0\Big]}.$$

Summing up what we have found above, we can state the main theorem as follows:

**Theorem 2.** *Every Bianchi surface in $E^3$ of class $C^4$ whose asymptotic lines are geodesic parallels is a helicoid or a surface of revolution and consequently, every such surface is applicable to some surface of revolution.*

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